## High School Mathematics in Middle School[4]

There are some students who are able to move through the mathematics quickly. These students may choose to take high school mathematics beginning in eighth grade[5] or earlier so they can take college-level mathematics in high school.[6] Students who are capable of moving more quickly deserve thoughtful attention, both to ensure that they are challenged and that they are mastering the full range of mathematical content and skills—without omitting critical concepts and topics. Care must be taken to ensure that students master and fully understand all important topics in the mathematics curriculum, and that the continuity of the mathematics learning progression is not disrupted. In particular, the Standards for Mathematical Practice ought to continue to be emphasized in these cases.

The number of students taking high school mathematics in eighth grade has increased steadily for years. Part of this trend is the result of a concerted effort to get more students to take Calculus and other college-level mathematics courses in high school. Enrollment in both AP Statistics and AP Calculus, for example, have essentially doubled over the last decade (College Board, 2009). There is also powerful research showing that among academic factors, the strongest predictor of whether a student will earn a bachelor's degree is the highest level of mathematics taken in high school (Adelman, 1999). A recent study completed by The College Board confirms this. Using data from 65,000 students enrolled in 110 colleges, students' high school coursework was evaluated to determine which courses were closely associated with students' successful performance in college. The study confirmed the importance of a rigorous curriculum throughout a students' high school career. Among other conclusions, the study found that students who took more advanced courses, such as Pre-Calculus in the 11th grade or Calculus in 12th grade, were more successful in college. Students who took AP Calculus at any time during their high school careers were most successful (Wyatt & Wiley, 2010). And even as more students are enrolled in more demanding courses, it does not necessarily follow that there must be a corresponding decrease in engagement and success (Cooney & Bottoms, 2009, p. 2).

At the same time, there are cautionary tales of pushing underprepared students into the first course of high school mathematics in the eighth grade. The Brookings Institute's 2009 Brown Center Report on American Education found that the NAEP scores of students taking Algebra I in the eighth grade varied widely, with the bottom ten percent scoring far below grade level. And a report from the Southern Regional Education Board, which supports increasing the number of middle students taking Algebra I, found that among students in the lowest quartile on achievement tests, those enrolled in higher-level mathematics had a slightly higher failure rate than those enrolled in lower-level mathematics (Cooney & Bottoms, 2009, p. 2). In all other quartiles, students scoring similarly on achievement tests were less likely to fail if they were enrolled in more demanding courses. These two reports are reminders that, rather than skipping or rushing through content, students should have appropriate progressions of foundational content to maximize their likelihoods of success in high school mathematics.

It is also important to note that notions of what constitutes a course called "Algebra I" or "Mathematics I" vary widely. In the CCSS, students begin preparing for algebra in Kindergarten, as they start learning about the properties of operations. Furthermore, much of the content central to typical Algebra I courses—namely linear equations, inequalities, and functions—is found in the 8$^{th}$ grade CCSS. The Algebra I course described here ("High School Algebra I"), however, is the first formal algebra course in the Traditional Pathway (concepts from this Algebra I course are developed across the first two courses of the integrated pathway). Enrolling an eighth-grade student in a watered down version of either the Algebra I course or Mathematics I course described here may in fact do students a disservice, as mastery of algebra including attention to the Standards for Mathematical Practice is fundamental for success in further mathematics and on college entrance examinations. As mentioned above, skipping material to get students to a particular point in the curriculum will likely create gaps in the students' mathematical background, which may create additional problems later, because students may be denied the opportunity for a rigorous Algebra I or Mathematics I course and may miss important content from eighth-grade mathematics.

## Middle School Acceleration

Taking the above considerations into account, as well as the recognition that there are other methods for accomplishing these goals, the Achieve Pathways Group endorses the notion that all students who are ready for rigorous high school mathematics in eighth grade should take such courses (Algebra I or Mathematics I), and that all middle schools should offer this opportunity to their students. To prepare students for high school mathematics in eighth grade, districts are encouraged to have a well-crafted sequence of **compacted courses**. The term "compacted" means to compress content, which requires a faster pace to complete, as opposed to skipping content. The Achieve Pathways Group has developed two compacted course sequences, one designed for districts using a traditional Algebra I – Geometry – Algebra II high school sequence, and the other for districts using an integrated sequence, which is commonly found internationally. Both are based on the idea that content should compact 3 years of content into 2 years, at most. In other words, compacting content from 2 years into 1 year would be too challenging, and compacting 4 years of content into 3 years starting in grade 7 runs the risk of compacting across middle and high schools. As such, grades 7, 8, and 9 were compacted into grades 7 and 8 (a 3:2 compaction). As a result, some 8$^{th}$ grade content is in the 7$^{th}$ grade courses, and high school content is in 8$^{th}$ grade.

[4]This section refers to mathematics content, not high school credit. The determination for high school credit is presumed to be made by state and local education agencies.
[5]Either 8th Grade Algebra I or Accelerated Mathematics I.
[6]Such as Calculus or Advanced Statistics.

The compacted traditional sequence, or, "Accelerated Traditional," compacts grades 7, 8, and High School Algebra I into two years: "Accelerated 7th Grade" and "8th Grade Algebra I." Upon successfully completion of this pathway, students will be ready for Geometry in high school. The compacted integrated sequence, or, "Accelerated Integrated," compacts grades 7, 8, and Mathematics I into two years: "Accelerated 7th Grade" and "8th Grade Mathematics I." At the end of 8th grade, these students will be ready for Mathematics II in high school. While the K-7 CCSS effectively prepare students for algebra in 8th grade, some standards from 8th grade have been placed in the Accelerated 7th Grade course to make the 8th Grade courses more manageable.

The Achieve Pathways Group has followed a set of guidelines[7] for the development of these compacted courses.

1. **Compacted courses should include the same Common Core State Standards as the non-compacted courses.** It is recommended to compact three years of material into two years, rather than compacting two years into one. The rationale is that mathematical concepts are likely to be omitted when trying to squeeze two years of material into one. This is to be avoided, as the standards have been carefully developed to define clear learning progressions through the major mathematical domains. Moreover, the compacted courses should not sacrifice attention to the Mathematical Practices Standard.

2. **Decisions to accelerate students into the Common Core State Standards for high school mathematics before ninth grade should not be rushed.** Placing students into tracks too early should be avoided at all costs. It is not recommended to compact the standards before grade seven. In this document, compaction begins in seventh grade for both the traditional and integrated (international) sequences.

3. **Decisions to accelerate students into high school mathematics before ninth grade should be based on solid evidence of student learning.** Research has shown discrepancies in the placement of students into "advanced" classes by race/ethnicity and socioeconomic background. While such decisions to accelerate are almost always a joint decision between the school and the family, serious efforts must be made to consider solid evidence of student learning in order to avoid unwittingly disadvantaging the opportunities of particular groups of students.

4. **A menu of challenging options should be available for students after their third year of mathematics—and all students should be strongly encouraged to take mathematics in all years of high school.** Traditionally, students taking high school mathematics in the eighth grade are expected to take Precalculus in their junior years and then Calculus in their senior years. This is a good and worthy goal, but it should not be the only option for students. Advanced courses could also include Statistics, Discrete Mathematics, or Mathematical Decision Making. An array of challenging options will keep mathematics relevant for students, and give them a new set of tools for their futures in college and career (see Fourth Courses section of this paper for further detail).

## Other Ways to Accelerate Students

Just as care should be taken not to rush the decision to accelerate students, care should also be taken to provide more than one opportunity for acceleration. Some students may not have the preparation to enter a "Compacted Pathway" but may still develop an interest in taking advanced mathematics, such as AP Calculus or AP Statistics in their senior year. Additional opportunities for acceleration may include:

- Allowing students to take two mathematics courses simultaneously (such as Geometry and Algebra II, or Precalculus and Statistics).
- Allowing students in schools with block scheduling to take a mathematics course in both semesters of the same academic year.
- Offering summer courses that are designed to provide the equivalent experience of a full course in all regards, including attention to the Mathematical Practices.[8]
- Creating different compaction ratios, including four years of high school content into three years beginning in 9th grade.
- Creating a hybrid Algebra II-Precalculus course that allows students to go straight to Calculus.

A combination of these methods and our suggested compacted sequences would allow for the most mathematically-inclined students to take advanced mathematics courses during their high school career. The compacted sequences begin here:

[7] Based on work published by Washington Office of the Superintendent of Public Schools, 2008
[8] As with other methods of accelerating students, enrolling students in summer courses should be handled with care, as the pace of the courses likely be enormously fast.

# Overview of the Accelerated Traditional Pathway for the Common Core State Mathematics Standards

This table shows the domains and clusters in each course in the Accelerated Traditional Pathway. The standards from each cluster included in that course are listed below each cluster. For each course, limits and focus for the clusters are shown in italics. For organizational purposes, clusters from 7th Grade and 8th Grade have been situated in the matrix within the high school domains.

| | Domains | Accelerated 7th Grade | 8th Grade Algebra I | Geometry | Algebra II | Fourth Courses* |
|---|---|---|---|---|---|---|
| Number and Quantity | The Real Number System | • Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.<br>7.NS.1a, 1b, 1c, 1d, 2a, 2b, 2c, 2d, 3<br>• Know that there are numbers that are not rational, and approximate them by rational numbers.<br>8.NS.1, 2<br>• Work with radicals and integer exponents.<br>8.EE.1, 2, 3, 4 | • Extend the properties of exponents to rational exponents.<br>N.RN.1, 2<br>• Use properties of rational and irrational numbers.<br>N.RN.3. | | | |
| | Quantities | • Analyze proportional relationships and use them to solve real-world and mathematical problems.<br>7.RP.1, 2a, 2b, 2c, 2d, 3 | • Reason quantitatively and use units to solve problems.<br>*Foundation for work with expressions, equations and functions*<br>N.Q.1, 2, 3 | | | |

*The (+) standards in this column are those in the Common Core State Standards that are not included in any of the Accelerated Traditional Pathway courses. They would be used in additional courses developed to follow Algebra II.

| | Domains | Accelerated 7th Grade | 8th Grade Algebra I | Geometry | Algebra II | Fourth Courses |
|---|---|---|---|---|---|---|
| Number and Quantity | The Complex Number System | | | | • Perform arithmetic operations with complex numbers.<br>N.CN.1, 2<br>• Use complex numbers in polynomial identities and equations.<br>*Polynomials with real coefficients*<br>N.CN.7, (+) 8, (+) 9 | • Perform arithmetic operations with complex numbers.<br>(+) N.CN.3<br>• Represent complex numbers and their operations on the complex plane.<br>(+) N.CN.4, 5, 6 |
| | Vector Quantities and Matrices | | | | | • Represent and model with vector quantities.<br>(+) N.VM.1, 2, 3<br>• Perform operations on vectors.<br>(+) N.VM.4a, 4b, 4c, 5a, 5b<br>• Perform operations on matrices and use matrices in applications.<br>(+) N.VM.6, 7, 8, 9, 10, 11, 12 |
| Algebra | Seeing Structure in Expressions | • Use properties of operations to generate equivalent expressions.<br>7.EE.1, 2<br>• Solve real-life and mathematical problems using numerical and algebraic expressions and equations..<br>7.EE.3, 4a, 4b | • Interpret the structure of expressions.<br>*Linear, exponential, quadratic*<br>A.SSE.1a, 1b, 2<br>• Write expressions in equivalent forms to solve problems.<br>*Quadratic and exponential*<br>A.SSE.3a, 3b, 3c | | • Interpret the structure of expressions.<br>*Polynomial and rational*<br>A.SSE.1a, 1b, 2<br>• Write expressions in equivalent forms to solve problems.<br>A.SSE.4 | |

| | Domains | Accelerated 7th Grade | 8th Grade Algebra I | Geometry | Algebra II | Fourth Courses |
|---|---|---|---|---|---|---|
| Algebra | Arithmetic with Polynomials and Rational Expressions | | • Perform arithmetic operations on polynomials.<br>*Linear and quadratic*<br>A.APR.1 | | • Perform arithmetic operations on polynomials.<br>*Beyond quadratic*<br>A.APR.1<br>• Understand the relationship between zeros and factors of polynomials.<br>A.APR.2, 3<br>• Use polynomial identities to solve problems.<br>A.APR.4, (+) 5<br>• Rewrite rational expressions.<br>*Linear and quadratic denominators*<br>A.APR.6, (+) 7 | |
| | Creating Equations | | • Create equations that describe numbers or relationships.<br>*Linear, quadratic, and exponential (integer inputs only) for A.CED.3, linear only*<br>A.CED. 1, 2, 3, 4 | | • Create equations that describe numbers or relationships.<br>*Equations using all available types of expressions, including simple root functions*<br>A.CED.1, 2, 3, 4 | |

| | Domains | Accelerated 7th Grade | 8th Grade Algebra I | Geometry | Algebra II | Fourth Courses |
|---|---|---|---|---|---|---|
| **Algebra** | Reasoning with Equations and Inequalities | • Understand the connections between proportional relationships, lines, and linear equations.<br>8.EE.5, 6<br>• Analyze and solve linear equations and pairs of simultaneous linear equations.<br>8.EE.7a, 7b | • Understand solving equations as a process of reasoning and explain the reasoning.<br>*Master linear, learn as general principle*<br>A.REI.1<br>• Solve equations and inequalities in one variable.<br>*Linear inequalities; literal equations that are linear in the variables being solved for; quadratics with real solutions*<br>A.REI.3, 4a, 4b<br>• Analyze and solve linear equations and pairs of simultaneous linear equations.<br>8.EE.8a, 8b, 8c<br>• Solve systems of equations.<br>*Linear-linear and linear-quadratic*<br>A.REI.5, 6, 7<br>• Represent and solve equations and inequalities graphically.<br>*Linear and exponential; learn as general principle*<br>A.REI.10, 11, 12 | | • Understand solving equations as a process of reasoning and explain the reasoning.<br>*Simple radical and rational*<br>A.REI.2<br>• Represent and solve equations and inequalities graphically.<br>*Combine polynomial, rational, radical, absolute value, and exponential functions*<br>A.REI.11 | • Solve systems of equations.<br>(+) A.REI.8, 9 |

| | Domains | Accelerated 7th Grade | 8th Grade Algebra I | Geometry | Algebra II | Fourth Courses |
|---|---|---|---|---|---|---|
| **Functions** | Interpreting Functions | | • Define, evaluate, and compare functions.<br>8.F.1, 2, 3<br>• Understand the concept of a function and use function notation.<br>*Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences*<br>F.IF.1, 2, 3<br>• Use functions to model relationships between quantities.<br>8.F.4, 5<br>• Interpret functions that arise in applications in terms of a context.<br>*Linear, exponential, and quadratic*<br>F.IF.4, 5, 6<br>• Analyze functions using different representations.<br>*Linear, exponential, quadratic, absolute value, step, piecewise-defined*<br>F.IF.7a, 7b, 7e, 8a, 8b, 9 | | • Interpret functions that arise in applications in terms of a context.<br>*Emphasize selection of appropriate models*<br>F.IF.4, 5, 6<br>• Analyze functions using different representations.<br>*Focus on using key features to guide selection of appropriate type of model function*<br>F.IF.7b, 7c, 7e, 8, 9 | • Analyze functions using different representations.<br>*Logarithmic and trigonometric functions*<br>(+) F.IF.7d |

| | Domains | Accelerated 7th Grade | 8th Grade Algebra I | Geometry | Algebra II | Fourth Courses |
|---|---|---|---|---|---|---|
| Functions | Building Functions | | • Build a function that models a relationship between two quantities.<br>*For F.BF.1, 2, linear, exponential, and quadratic*<br>F.BF.1a, 1b, 2<br>• Build new functions from existing functions.<br>*Linear, exponential, quadratic, and absolute value; for F.BF.4a, linear only*<br>F.BF.3, 4a | | • Build a function that models a relationship between two quantities.<br>*Include all types of functions studied*<br>F.BF.1b<br>• Build new functions from existing functions.<br>*Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types*<br>F.BF.3, 4a | • Build a function that models a relationship between two quantities.<br>(+) F.BF.1c<br>• Build new functions from existing functions.<br>(+) F.BF.4b, 4c, 4d, 5 |
| | Linear, Quadratic, and Exponential Models | | • Construct and compare linear, quadratic, and exponential models and solve problems.<br>F.LE.1a, 1b, 1c, 2, 3<br>• Interpret expressions for functions in terms of the situation they model.<br>*Linear and exponential of form $f(x) = b^x + k$*<br>F.LE.5 | | • Construct and compare linear, quadratic, and exponential models and solve problems.<br>*Logarithms as solutions for exponentials*<br>F.LE.4 | |
| | Trigonometric Functions | | | | • Extend the domain of trigonometric functions using the unit circle.<br>F.TF.1, 2<br>• Model periodic phenomena with trigonometric functions.<br>F.TF.5<br>• Prove and apply trigonometric identities.<br>F.TF.8 | • Extend the domain of trigonometric functions using the unit circle.<br>(+) F.TF.3, 4<br>• Model periodic phenomena with trigonometric functions.<br>(+) F.TF. 6, 7<br>• Prove and apply trigonometric identities.<br>(+) F.TF. 9 |

| | Domains | Accelerated 7th Grade | 8th Grade Algebra I | Geometry | Algebra II | Fourth Courses |
|---|---|---|---|---|---|---|
| **Geometry** | Congruence | • Draw, construct, and describe geometrical figures and describe the relationships between them.<br>*Focus on constructing triangles*<br>7.G.2<br><br>• Understand congruence and similarity using physical models, transparencies, or geometric software.<br>8.G.1a, 1b, 1c, 2, 5<br><br>• For 8.G.5, informal arguments to establish angle sum and exterior angle theorems for triangles and angles relationships when parallel lines are cut by a transversal | | • Experiment with transformations in the plane.<br>G.CO.1, 2, 3, 4, 5<br><br>• Understand congruence in terms of rigid motions.<br>*Build on rigid motions as a familiar starting point for development of concept of geometric proof*<br>G.CO.6, 7, 8<br><br>• Prove geometric theorems.<br>*Focus on validity of underlying reasoning while using variety of ways of writing proofs*<br>G.CO.9, 10, 11<br><br>• Make geometric constructions.<br>*Formalize and explain processes*<br>G.CO.12,13 | | |
| | Similarity, Right Tri-angles, and Trigonom-etry | • Draw, construct, and describe geometrical figures and describe the relationships between them.<br>*Scale drawings*<br>7.G.1<br><br>• Understand congruence and similarity using physical models, transparencies, or geometric software.<br>8.G.3, 4, 5<br><br>• For 8.G.5, informal arguments to establish the angle-angle criterion for similar triangles | | • Understand similarity in terms of similarity transformations.<br>G.SRT.1a, 1b, 2, 3<br><br>• Prove theorems involving similarity.<br>G.SRT.4, 5<br><br>• Define trigonometric ratios and solve problems involving right triangles.<br>G.SRT.6, 7, 8<br><br>• Apply trigonometry to general triangles.<br>G.SRT.9. 10, 11 | | |

| | Domains | Accelerated 7th Grade | 8th Grade Algebra I | Geometry | Algebra II | Fourth Courses |
|---|---|---|---|---|---|---|
| Geometry | Circles | | | • Understand and apply theorems about circles.<br>G.C.1, 2, 3, (+) 4<br>• Find arc lengths and areas of sectors of circles.<br>*Radian introduced only as unit of measure*<br>G.C.5 | | |
| | Expressing Geometric Properties with Equations | | | • Translate between the geometric description and the equation for a conic section.<br>G.GPE.1, 2<br>• Use coordinates to prove simple geometric theorems algebraically.<br>*Include distance formula; relate to Pythagorean theorem*<br>G.GPE. 4, 5, 6, 7 | | • Translate between the geometric description and the equation for a conic section.<br>(+) G.GPE.3 |
| | Geometric Measurement and Dimension | • Draw, construct, and describe geometrical figures and describe the relationships between them.<br>*Slicing 3-D figures*<br>7.G.3<br>• Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.<br>7.G.4, 5, 6<br>• Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.<br>8.G.9 | • Understand and apply the Pythagorean theorem.<br>*Connect to radicals, rational exponents, and irrational numbers*<br>8.G.6, 7, 8 | • Explain volume formulas and use them to solve problems.<br>G.GMD.1, 3<br>• Visualize the relation between two-dimensional and three-dimensional objects.<br>G.GMD.4 | | • Explain volume formulas and use them to solve problems.<br>(+) G.GMD.2 |
| | Modeling with Geometry | | | • Apply geometric concepts in modeling situations.<br>G.MG.1, 2, 3 | | |

| | Domains | Accelerated 7th Grade | 8th Grade Algebra I | Geometry | Algebra II | Fourth Courses |
|---|---|---|---|---|---|---|
| **Statistics and Probability** | Interpreting Categorical and Quantitative Data | | • Summarize, represent, and interpret data on a single count or measurement variable.<br>S.ID.1, 2, 3<br>• Investigate patterns of association in bivariate data.<br>8.SP.1, 2, 3, 4<br>• Summarize, represent, and interpret data on two categorical and quantitative variables.<br>*Linear focus; discuss general principle*<br>S.ID.5, 6a, 6b, 6c<br>• Interpret linear models.<br>S.ID.7, 8, 9 | | • Summarize, represent, and interpret data on a single count or measurement variable.<br>S.ID.4 | |
| | Making Inferences and Justifying Conclusions | • Use random sampling to draw inferences about a population.<br>7.SP.1, 2<br>• Draw informal comparative inferences about two populations.<br>7.SP.3, 4 | | | • Understand and evaluate random processes underlying statistical experiments.<br>S.IC.1, 2<br>• Make inferences and justify conclusions from sample surveys, experiments and observational studies.<br>S.IC.3, 4, 5, 6 | |

| | Domains | Accelerated 7th Grade | 8th Grade Algebra I | Geometry | Algebra II | Fourth Courses |
|---|---|---|---|---|---|---|
| Statistics and Probability | Conditional Probability and the Rules of Probability | • Investigate chance processes and develop, use, and evaluate probability models.<br>7.SP.5, 6, 7a, 7b, 8a, 8b, 8c | | • Understand independence and conditional probability and use them to interpret data.<br>*Link to data from simulations or experiments*<br>S.CP.1, 2, 3, 4, 5<br>• Use the rules of probability to compute probabilities of compound events in a uniform probability model.<br>S.CP.6, 7, (+) 8, (+) 9 | | |
| | Using Probability to Make Decisions | | | • Use probability to evaluate outcomes of decisions.<br>*Introductory; apply counting rules*<br>(+) S.MD.6, 7 | • Use probability to evaluate outcomes of decisions.<br>*Include more complex situations*<br>(+) S.MD.6, 7 | • Calculate expected values and use them to solve problems.<br>(+) S.MD.1, 2, 3, 4<br>• Use probability to evaluate outcomes of decisions.<br>(+) S.MD. 5a, 5b |

# Accelerated Traditional Pathway: Accelerated 7$^{th}$ Grade

This course differs from the non-accelerated 7$^{th}$ Grade course in that it contains content from 8$^{th}$ grade. While coherence is retained, in that it logically builds from the 6$^{th}$ Grade, the additional content when compared to the non-accelerated course demands a faster pace for instruction and learning. Content is organized into four critical areas, or units. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas are as follows:

Critical Area 1: Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems. They extend their mastery of the properties of operations to develop an understanding of integer exponents, and to work with numbers written in scientific notation.

Critical Area 2: Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality ($m$) is the slope, and the graphs are lines through the origin. They understand that the slope ($m$) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \times A$. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation.

Critical Area 3: Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Critical Area 4: Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity, they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

<table>
<tr><th>Units</th><th>Includes Standard Clusters*</th><th>Mathematical Practice Standards</th></tr>
<tr><td>Unit 1<br>Rational Numbers and Exponents</td><td>• Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.<br>• Know that there are numbers that are not rational, and approximate them by rational numbers.<br>• Work with radicals and integer exponents.</td><td rowspan="4">Make sense of problems and persevere in solving them.<br><br>Reason abstractly and quantitatively.<br><br>Construct viable arguments and critique the reasoning of others.<br><br>Model with mathematics.<br><br>Use appropriate tools strategically.<br><br>Attend to precision.<br><br>Look for and make use of structure.<br><br>Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Unit 2<br>Proportionality and LInear Relationships</td><td>• Analyze proportional relationships and use them to solve real-world and mathematical problems.<br>• Use properties of operations to generate equivalent expressions.<br>• Solve real-life and mathematical problems using numerical and algebraic expressions and equations.<br>• Understand the connections between proportional relationships, lines, and linear equations.<br>• Analyze and solve linear equations and pairs of simultaneous linear equations.</td></tr>
<tr><td>Unit 3<br>Introduction to Sampling Inference</td><td>• Use random sampling to draw inferences about a population.<br>• Draw informal comparative inferences about two populations.<br>• Investigate chance processes and develop, use, and evaluate probability models.</td></tr>
<tr><td>Unit 4<br>Creating, Comparing, and Analyzing Geometric Figures</td><td>• Draw, construct and describe geometrical figures and describe the relationships between them.<br>• Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.<br>• Understand congruence and similarity using physical models, transparencies, or geometry software.<br>• Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.</td></tr>
</table>

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

## Unit 1: Rational Numbers and Exponents

Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems. They extend their mastery of the properties of operations to develop an understanding of integer exponents, and to work with numbers written in scientific notation.

| **Unit 1: Rational Numbers and Exponents** | |
|---|---|
| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. | 7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.<br>a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*<br>b. Understand $p + q$ as the number located a distance $\|q\|$ from $p$, in the positive or negative direction depending on whether $q$ is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.<br>c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.<br>d. Apply properties of operations as strategies to add and subtract rational numbers.<br>7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.<br>a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.<br>b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If $p$ and $q$ are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.<br>c. Apply properties of operations as strategies to multiply and divide rational numbers.<br>d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.<br>7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.* |

*Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

| Unit 1: Rational Numbers and Exponents | |
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| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Know that there are numbers that are not rational, and approximate them by rational numbers. | 8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.<br><br>8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., p2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.* |
| • Work with radicals and integer exponents. | 8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.<br><br>8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.<br><br>8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* $3 \times 10^8$ *and the population of the world as* $7 \times 10^9$*, and determine that the world population is more than 20 times larger.*<br><br>8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. |

## Unit 2: Proportionality and Linear Relationships

Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality ($m$) is the slope, and the graphs are lines through the origin. They understand that the slope ($m$) of a line is a constant rate of change, so that if the input or $x$-coordinate changes by an amount $A$, the output or $y$-coordinate changes by the amount $m \times A$. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation.

| **Unit 2: Proportionality and Linear Relationships** | |
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| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Analyze proportional relationships and use them to solve real-world and mathematical problems. | 7.RP.1 *Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks* $^1/_2$ *mile in each* $^1/_4$ *hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.*<br><br>7.RP.2 *Recognize and represent proportional relationships between quantities.*<br>a. *Decide whether two quantities are in a proportional relationship,* e.g., by testin*g for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.*<br>b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.<br>c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*<br>d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.<br><br>7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* |
| • Use properties of operations to generate equivalent expressions. | 7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.<br><br>7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."* |

| Unit 2: Proportionality and Linear Relationships | |
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| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Solve real-life and mathematical problems using numerical and algebraic expressions and equations. | 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*<br><br>*7.EE.4* Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.<br>a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where $p$, $q$, and $r$ are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*<br>b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where $p$, $q$, and $r$ are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.* |
| • Understand the connections between proportional relationships, lines, and linear equations. | 8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*<br><br>8.EE.6 Use similar triangles to explain why the slope $m$ is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $b$. |
| • Analyze and solve linear equations and pairs of simultaneous linear equations. | 8.EE.7 Solve linear equations in one variable.<br>a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where $a$ and $b$ are different numbers).<br>b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. |

## Unit 3: Introduction to Sampling and Inference

Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

| Unit 3: Introduction to Sampling and Inference | |
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| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Use random sampling to draw inferences about a population. | 7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.<br><br>7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* |
| • Draw informal comparative inferences about two populations. | 7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*<br><br>7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* |

| Unit 3: Introduction to Sampling and Inference | |
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| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Investigate chance processes and develop, use, and evaluate probability models. | 7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $^1/_2$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.<br><br>7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*<br><br>7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.<br>a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*<br>b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*<br><br>7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.<br>a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.<br>b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.<br>c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?* |

## Unit 4: Creating, Comparing, and Analyzing Geometric Figures

Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity, they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

| Unit 4: Creating, Comparing, and Analyzing Geometric Figures | |
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| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Draw, construct, and describe geometrical figures and describe the relationships between them. | 7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.<br><br>7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.<br><br>7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. |
| • Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. | 7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.<br><br>7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.<br><br>7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. |

| Unit 4: Creating, Comparing, and Analyzing Geometric Figures | |
|---|---|
| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Understand congruence and similarity using physical models, transparencies, or geometry software. | 8.G.1 Verify experimentally the properties of rotations, reflections, and translations:<br>a. Lines are taken to lines, and line segments to line segments of the same length.<br>b. Angles are taken to angles of the same measure.<br>c. Parallel lines are taken to parallel lines.<br>8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.<br>8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.<br>8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.<br>8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.* |
| • Solve real-world and mathematical problem involving volume of cylinders, cones, and spheres. | 8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. |

# 8th Grade Algebra I

The fundamental purpose of 8th Grade Algebra I is to formalize and extend the mathematics that students learned through the end of seventh grade. The critical areas, called units, deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. In addition, the units will introduce methods for analyzing and using quadratic functions, including manipulating expressions for them, and solving quadratic equations. Students understand and apply the Pythagorean theorem, and use quadratic functions to model and solve problems. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

This course differs from High School Algebra I in that it contains content from 8th grade. While coherence is retained, in that it logically builds from the Accelerated 7th Grade, the additional content when compared to the high school course demands a faster pace for instruction and learning.

Critical Area 1: Work with quantities and rates, including simple linear expressions and equations forms the foundation for this unit. Students use units to represent problems algebraically and graphically, and to guide the solution of problems. Student experience with quantity provides a foundation for the study of expressions, equations, and functions. This unit builds on earlier experiences with equations by asking students to analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

Critical Area 2: Building on earlier work with linear relationships, students learn function notation and language for describing characteristics of functions, including the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students explore systems of equations and inequalities, and they find and interpret their solutions. Students build on and informally extend their understanding of integral exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3: Students use regression techniques to describe relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area 4: In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

Critical Area 5: In preparation for work with quadratic relationships students explore distinctions between rational and irrational numbers. They consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows $x+1 = 0$ to have a solution. Formal work with complex numbers comes in Algebra II. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

<table>
<tr><th>Units</th><th>Includes Standard Clusters*</th><th>Mathematical Practice Standards</th></tr>
<tr><td>Unit 1<br>Relationships Between Quantities and Reasoning with Equations</td><td>• Reason quantitatively and use units to solve problems.<br>• Interpret the structure of expressions.<br>• Create equations that describe numbers or relationships.<br>• Understand solving equations as a process of reasoning and explain the reasoning.<br>• Solve equations and inequalities in one variable.</td><td rowspan="3">Make sense of problems and persevere in solving them.<br><br>Reason abstractly and quantitatively.<br><br>Construct viable arguments and critique the reasoning of others.<br><br>Model with mathematics.<br><br>Use appropriate tools strategically.<br><br>Attend to precision.<br><br>Look for and make use of structure.<br><br>Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Unit 2<br>Linear and Exponential Relationships</td><td>• Extend the properties of exponents to rational exponents.<br>• Analyze and solve linear equations and pairs of simultaneous linear equations.<br>• Solve systems of equations.<br>• Represent and solve equations and inequalities graphically<br>• Define, evaluate, and compare functions.<br>• Understand the concept of a function and use function notation.<br>• Use functions to model relationships between quantities.<br>• Interpret functions that arise in applications in terms of a context.<br>• Analyze functions using different representations.<br>• Build a function that models a relationship between two quantities.<br>• Build new functions from existing functions.<br>• Construct and compare linear, quadratic, and exponential models and solve problems.<br>• Interpret expressions for functions in terms of the situation they model.</td></tr>
<tr><td>Unit 3<br>Descriptive Statistics</td><td>• Summarize, represent, and interpret data on a single count or measurement variable.<br>• Investigate patterns of association in bivariate data.<br>• Summarize, represent, and interpret data on two categorical and quantitative variables.<br>• Interpret linear models.</td></tr>
</table>

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

| Units | Includes Standard Clusters | Mathematical Practice Standards |
|---|---|---|
| **Unit 4**<br>Expressions and Equations | • Interpret the structure of expressions.<br>• Write expressions in equivalent forms to solve problems.<br>• Perform arithmetic operations on polynomials.<br>• Create equations that describe numbers or relationships.<br>• Solve equations and inequalities in one variable.<br>• Solve systems of equations. | |
| **Unit 5**<br>Quadratics Funtions and Modeling | • Use properties of rational and irrational numbers.<br>• Understand and apply the Pythagorean theorem.<br>• Interpret functions that arise in applications in terms of a context.<br>• Analyze functions using different representations.<br>• Build a function that models a relationship between two quantities.<br>• Build new functions from existing functions.<br>• Construct and compare linear, quadratic and exponential models and solve problems. | |

## Unit 1: Relationships between Quantities and Reasoning with Equations

Work with quantities and rates, including simple linear expressions and equations forms the foundation for this unit. Students use units to represent problems algebraically and graphically, and to guide the solution of problems. Student experience with quantity provides a foundation for the study of expressions, equations, and functions. This unit builds on earlier experiences with equations by asking students to analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

| **Unit 1: Relationships between Quantities and Reasoning with Equations** | |
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| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Reason quantitatively and use units to solve problems.<br><br>*Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.*[12] | N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.<br><br>N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.<br><br>N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. |
| • Interpret the structure of expressions.<br><br>*Limit to linear expressions and to exponential expressions with integer exponents.* | A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★<br><br>a. Interpret parts of an expression, such as terms, factors, and coefficients.<br><br>b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* |
| • Create equations that describe numbers or relationships.<br><br>*Limit A.CED.1 and A.CED.2 to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. Limit A.CED.3 to linear equations and inequalities. Limit A.CED.4 to formulas which are linear in the variables of interest.* | A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*<br><br>A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.<br><br>A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*<br><br>A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.* |
| • Understand solving equations as a process of reasoning and explain the reasoning.<br><br>*Students should focus on and master A.REI.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future units and courses. Students will solve exponential equations in Algebra II.* | A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. |

| Unit 1: Relationships between Quantities and Reasoning with Equations | |
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| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Solve equations and inequalities in one variable.<br><br>*Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = \frac{1}{16}$.* | A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. |

## Unit 2: Linear and Exponential Functions

Building on earlier work with linear relationships, students learn function notation and language for describing characteristics of functions, including the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students explore systems of equations and inequalities, and they find and interpret their solutions. Students build on and informally extend their understanding of integral exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

| Unit 2: Linear and Exponential Functions | |
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| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Extend the properties of exponents to rational exponents.<br><br>*In implementing the standards in curriculum, these standards should occur before discussing exponential models with continuous domains.* | N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*<br><br>N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. |
| • Analyze and solve linear equations and pairs of simultaneous linear equations.<br><br>*While this content is likely subsumed by A.REI.3, 5, and 6, it could be used for scaffolding instruction to the more sophisticated content found there.* | 8.EE.8 Analyze and solve pairs of simultaneous linear equations.<br>a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.<br>b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*<br>c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* |
| • Solve systems of equations.<br><br>*Include cases where two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution).* | A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.<br><br>A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. |

| Unit 2: Linear and Exponential Functions | |
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| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Represent and solve equations and inequalities graphically.<br><br>*For A.REI.10 focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses. For A.REI.11, focus on cases where f(x) and g(x) are linear or exponential.* | A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).<br><br>A.REI.11 Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★<br><br>A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. |
| • Define, evaluate, and compare functions.<br><br>*While this content is likely subsumed by F.IF.1-3 and F.IF.7a, it could be used for scaffolding instruction to the more sophisticated content found there.* | 8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.<br><br>8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*<br><br>8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.* |
| • Understand the concept of a function and use function notation.<br><br>*Students should experience a variety of types of situations modeled by functions Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses.*<br><br>*Constrain examples to linear functions and exponential functions having integral domains. In F.IF.3, draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences.* | F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and x is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.<br><br>F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.<br><br>F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$. |
| • Use functions to model relationships between quantities.<br><br>*While this content is likely subsumed by F.IF.4 and F.BF.1a, it could be used for scaffolding instruction to the more sophisticated content found there.* | 8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two ($x$, $y$) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.<br><br>8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. |

| Unit 2: Linear and Exponential Functions | |
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| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Interpret functions that arise in applications in terms of a context.<br><br>*For F.IF.4 and 5, focus on linear and exponential functions. For F.IF.6, focus on linear functions and exponential functions whose domain is a subset of the integers. Unit 5 in this course and Algebra II course address other types of functions.* | F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★<br><br>F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★<br><br>F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ |
| • Analyze functions using different representations.<br><br>*For F.IF.7a, 7e, and 9 focus on linear and exponentials functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y=3^n$ and $y=100\cdot2^n$.* | F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★<br>a. Graph linear and quadratic functions and show intercepts, maxima, and minima.<br>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.<br><br>F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. |
| • Build a function that models a relationship between two quantities.<br><br>*Limit F.BF.1a, 1b, and 2 to linear and exponential functions. In F.BF.2, connect arithmetic sequences to linear functions and geometric sequences to exponential functions in F.BF.2.* | F.BF.1 Write a function that describes a relationship between two quantities.★<br>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.<br>b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*<br><br>F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★ |
| • Build new functions from existing functions.<br><br>*Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.*<br><br>*While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.* | F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* |

| Unit 2: Linear and Exponential Functions | |
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| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Construct and compare linear, quadratic, and exponential models and solve problems.<br><br>*For F.LE.3, limit to comparisons between linear and exponential models.* | F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.<br>a. Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals.<br>b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.<br>c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.<br><br>F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).<br><br>F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. |
| • Interpret expressions for functions in terms of the situation they model.<br><br>*Limit exponential functions to those of the form $f(x) = b^x + k$.* | F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. |

## Unit 3: Descriptive Statistics

Students use regression techniques to describe relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

| Unit 3: Descriptive Statistics | |
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| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Summarize, represent, and interpret data on a single count or measurement variable.<br><br>*In grades 6 – 7, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.* | S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).<br><br>S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.<br><br>S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). |
| • Investigate patterns of association in bivariate data.<br><br>*While this content is likely subsumed by S.ID.6-9, it could be used for scaffolding instruction to the more sophisticated content found there.* | 8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.<br><br>8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.<br><br>8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*<br><br>8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?* |
| • Summarize, represent, and interpret data on two categorical and quantitative variables.<br><br>*Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.*<br><br>*S.ID.6b should be focused on linear models, but may be used to preface quadratic functions in the Unit 6 of this course.* | S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.<br><br>S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.<br>a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.*<br>b. Informally assess the fit of a function by plotting and analyzing residuals.<br>c. Fit a linear function for a scatter plot that suggests a linear association. |

| Unit 3: Descriptive Statistics | |
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| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Interpret linear models.<br><br>*Build on students' work with linear relationship and; introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure how well the data fit the relationship. The important distinction between a statistical relationship and a cause-and-effect relationship arises in S.ID.9.* | S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.<br><br>S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.<br><br>S.ID.9 Distinguish between correlation and causation. |

## Unit 4: Expressions and Equations

In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

| Unit 4: Expressions and Equations | |
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| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Interpret the structure of expressions.<br><br>*Focus on quadratic and exponential expressions. For A.SSE.1b, exponents are extended from integer found in Unit 1 to rational exponents focusing on those that represent square roots and cube roots.* | A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★<br>a. Interpret parts of an expression, such as terms, factors, and coefficients.<br>b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*<br><br>A.SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* |
| • Write expressions in equivalent forms to solve problems.<br><br>*Consider extending this unit to include the relationship between properties of logarithms and properties of exponents.* | A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★<br>a. Factor a quadratic expression to reveal the zeros of the function it defines.<br>b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.<br>c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression $1.15^t$ can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* |
| • Perform arithmetic operations on polynomials.<br><br>*Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of* x. | A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. |
| • Create equations that describe numbers or relationships.<br><br>*Extend work on linear and exponential equations in Unit 1 to include quadratic equations. Extend A.CED.4 to formulas involving squared variables.* | A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*<br><br>A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.<br><br>A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.* |

| Unit 4: Expressions and Equations | |
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| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Solve equations and inequalities in one variable.<br><br>*Students should learn of the existence of the complex number system, but will not solve quadratics with complex solutions until Algebra II.* | A.REI.4 Solve quadratic equations in one variable.<br>a. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.<br>b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$. |
| • Solve systems of equations.<br><br>*Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between $x^2+y^2=1$ and $y = (x+1)/2$ leads to the point $(3/5, 4/5)$ on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.* | A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.* |

## Unit 5: Quadratic Functions and Modeling

In preparation for work with quadratic relationships students explore distinctions between rational and irrational numbers. They consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows $x+1 = 0$ to have a solution. Formal work with complex numbers comes in Algebra II. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

| Unit 5: Quadratic Functions and Modeling | |
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| **Clusters with Instructional Notes** | **Common Core State Standards** |
| • Use properties of rational and irrational numbers.<br><br>*Connect N.RN.3 to physical situations, e.g., finding the perimeter of a square of area 2.* | N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. |
| • Understand and apply the Pythagorean theorem.<br><br>*Discuss applications of the Pythagorean theorem and its connections to radicals, rational exponents, and irrational numbers.* | 8.G.6 Explain a proof of the Pythagorean theorem and its converse.<br><br>8.G.7 Apply the Pythagorean theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.<br><br>8.G.8 Apply the Pythagorean theorem to find the distance between two points in a coordinate system. |
| • Interpret functions that arise in applications in terms of a context.<br><br>*Focus on quadratic functions; compare with linear and exponential functions studied in Unit 2.* | F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★<br><br>F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★<br><br>F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ |