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GRADE 3 • MODULE 4

Multiplication and Area

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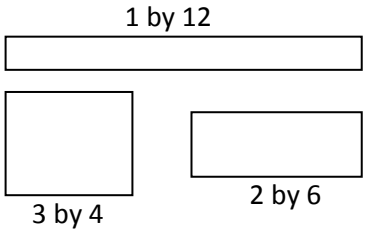
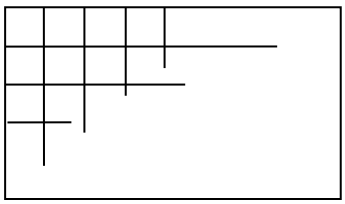
OVERVIEW

In this 20-day module students explore area as an attribute of two-dimensional figures and relate it to their prior understandings of multiplication. In Grade 2, students partitioned a rectangle into rows and columns of same-sized squares and found the total number by both counting and adding equal addends represented by the rows or columns (**2.G.2**, **2.OA.4**).

In Topic A, students begin to conceptualize area as the amount of two-dimensional surface that is contained within a plane figure. They come to understand that the space can be tiled with unit squares without gaps or overlaps (**3.MD.5**). They make predictions and explore which rectangles cover the most area when the side lengths differ (but area is actually the same). Students may, for example, cut and fold rectangles to confirm predictions about whether a 1 by 12 rectangle covers more area than a 3 by 4 or a 2 by 6 rectangle. They reinforce their ideas by using inch and centimeter square manipulatives to tile the same rectangles and prove the areas are equal. Topic A provides students' first experience with tiling, from which they learn to distinguish between length and area by placing a ruler with the same size units (inches or centimeters) next to a tiled array to discover that the number of tiles along a side corresponds to the length of the side (**3.MD.6**).

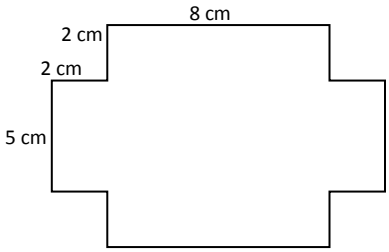


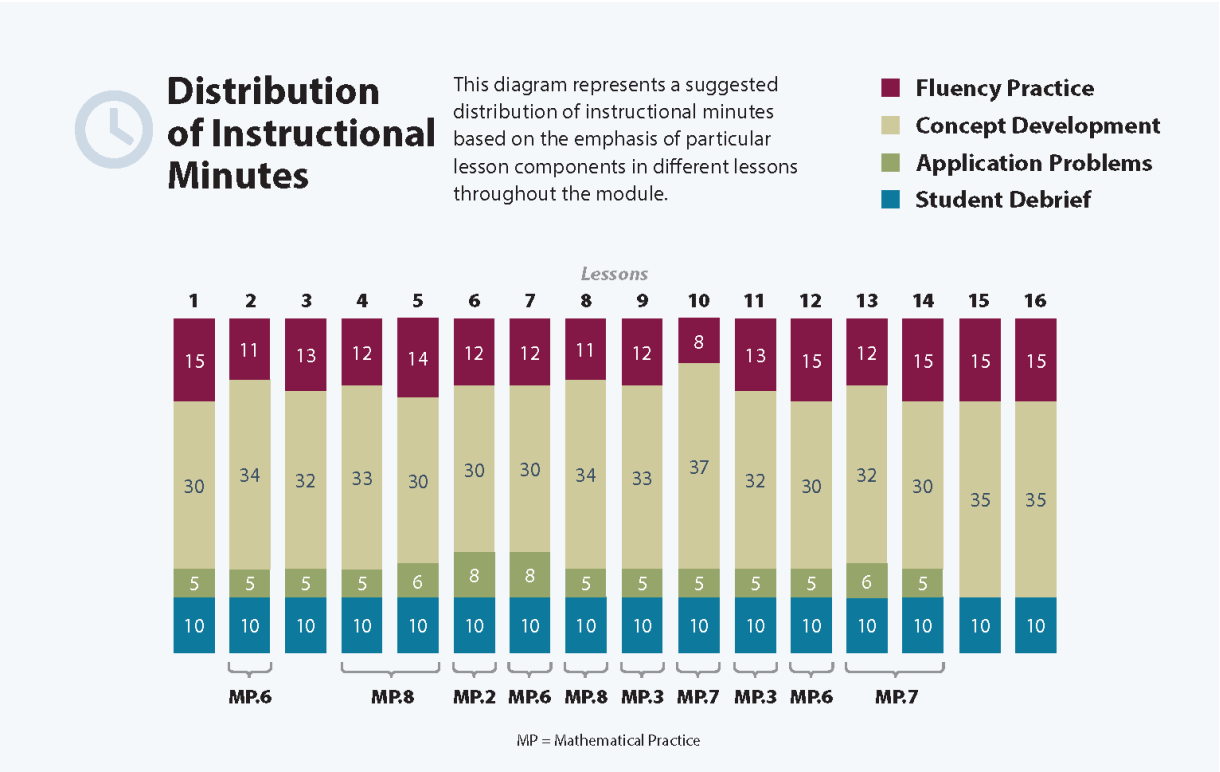
In Topic B, students progress from using square tile manipulatives to drawing their own area models. Anticipating the final structure of an array, they complete rows and columns in figures such as the example shown at the right. Students connect their extensive work with rectangular arrays and multiplication to eventually discover the area formula for a rectangle, which is formally introduced in Grade 4 (**3.MD.7a**).



In Topic C, students manipulate rectangular arrays to concretely demonstrate the arithmetic properties in anticipation of the following lessons. They do this by cutting rectangular grids and rearranging the parts into new wholes using the properties to validate that area stays the same, despite the new dimensions. They apply tiling and multiplication skills to determine all whole number possibilities for the side lengths of rectangles given their areas (**3.MD.7b**).

Topic D creates an opportunity for students to solve problems involving area (**3.MD.7b**). Students decompose and/or compose composite regions like the one shown at right into non-overlapping rectangles, find the area of each region, and add or subtract to determine the total area of the original shape. This leads students to design a simple floor plan that conforms to given area specifications (**3.MD.7d**).





Focus Grade Level Standards

Geometric Measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement:

a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

3.MD.7 Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

d. Recognize area as additive. Find the areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Foundational Standards

2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively**. Students build toward abstraction starting with tiling a rectangle, then gradually moving to finishing incomplete grids and drawing grids of their own, then eventually working purely in the abstract, imaging the grid as needed.

MP.3 **Construct viable arguments and critique the reasoning of others**. Students explore their conjectures about area by cutting to decompose rectangles and then recomposing them in different ways to determine if different rectangles have the same area. When solving area problems, students learn to justify their reasoning and determine whether they have found all possible solutions, when multiple solutions are possible.

MP.6 **Attend to precision**. Students precisely label models and interpret them, recognizing that the unit impacts the amount of space a particular model represents, even though pictures may appear to show equal sized models. They understand why when side lengths are multiplied the result is given in square units.

MP.7 **Look for and make use of structure**. Students relate previous knowledge of the commutative and distributive properties to area models. They build from spatial structuring to understanding the number of area-units as the product of number of units in a row and number of rows.

MP.8 **Look for and express regularity in repeated reasoning.** Students use increasingly sophisticated strategies to determine area over the course of the module. As they analyze and compare strategies, they eventually realize that area can be found by multiplying the number in each row by the number of rows.

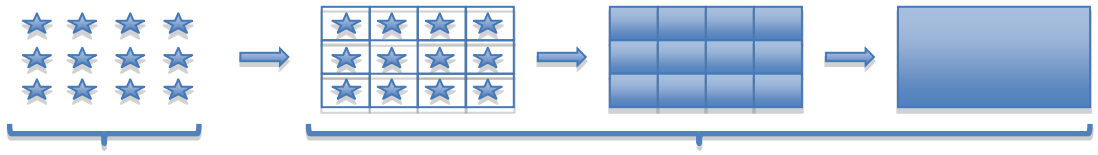
Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives		Days
3.MD.5 3.MD.6 3.MD.7	A	**Foundations for Understanding Area** Lesson 1: Understand area as an attribute of plane figures. Lesson 2: Decompose and recompose shapes to compare areas. Lesson 3: Model tiling with centimeter and inch unit squares as a strategy to measure area. Lesson 4: Relate side lengths with the number of tiles on a side.	4
3.MD.5 **3.MD.6** **3.MD.7a** **3.MD.7b** **3.MD.7d**	B	**Concepts of Area Measurement** Lesson 5: Form rectangles by tiling with unit squares to make arrays. Lesson 6: Draw rows and columns to determine the area of a rectangle, given an incomplete array. Lesson 7: Interpret area models to form rectangular arrays. Lesson 8: Find the area of a rectangle through multiplication of the side lengths.	4
		Mid-Module Assessment: Topics A–B (assessment 1 day, return ½ day, remediation or further applications ½ day)	2
3.MD.5 **3.MD.6** **3.MD.7a** **3.MD.7b** **3.MD.7c** **3.MD.7d**	C	**Arithmetic Properties Using Area Models** Lesson 9: Analyze different rectangles and reason about their area. Lesson 10: Apply the distributive property as a strategy to find the total area of a large rectangle by adding two products. Lesson 11: Demonstrate the possible whole number side lengths of rectangles with areas of 24, 36, 48, or 72 square units using the associative property.	3
3.MD.6 **3.MD.7a** **3.MD.7b** **3.MD.7c** **3.MD.7d** 3.MD.5	D	**Applications of Area Using Side Lengths of Figures** Lesson 12: Solve word problems involving area. Lessons 13–14: Find areas by decomposing into rectangles or completing composite figures to form rectangles. Lessons 15–16: Apply knowledge of area to determine areas of rooms in a given floor plan.	5
		End-of-Module Assessment: Topics A–D (assessment 1 day, return ½ day, remediation or further applications ½ day)	2
Total Number of Instructional Days			**20**

Terminology

New or Recently Introduced Terms

- Area (the amount of two-dimensional space in a bounded region)
- Area model (a model for multiplication that relates rectangular arrays to area)



- Square unit (a unit of area—specifically square centimeters, inches, feet, and meters)
- Tile (to cover a region without gaps or overlaps)
- Unit square (e.g., given a length unit, it is a 1 unit by 1 unit square)
- Whole number (an integer, a number without fractions)

Familiar Terms and Symbols[1]

- Array (a set of numbers or objects that follow a specific pattern, a matrix)
- Commutative Property (e.g., rotate a rectangular array 90 degrees to demonstrate that factors in a multiplication sentence can switch places)
- Distribute (e.g., $2 \times (3 + 4) = 2 \times 3 + 2 \times 4$)
- Geometric shape (a two-dimensional object with a specific outline or form)
- Length (the straight-line distance between two points)
- Multiplication (e.g., $5 \times 3 = 15$)
- Rows and columns (e.g., in reference to rectangular arrays)

Suggested Tools and Representations

- Area model
- Array
- Grid paper (inch and centimeter)
- Rulers (both centimeter and inch measurements)
- Unit squares in both inch and centimeter lengths (e.g., square tiles used for measuring area)

[1] These are terms and symbols students have seen previously.

Scaffolds[2]

The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units,* please refer to "How to Implement *A Story of Units*."

Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	3.MD.5 3.MD.6 3.MD.7abd
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	3.MD.5 3.MD.6 3.MD.7a–d

[2] Students with disabilities may require Braille, large print, audio, or special digital files. Please visit the website, www.p12.nysed.gov/specialed/aim, for specific information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.

Topic A

Foundations for Understanding Area

3.MD.5, 3.MD.6, 3.MD.7

Focus Standard:	3.MD.5	Recognize area as an attribute of plane figures and understand concepts of area measurement: a. A square with side length 1 unit, called a "square unit," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.
Instructional Days:	4	
Coherence -Links from:	G2–M2	Addition and Subtraction of Length Units
	G3–M1	Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10
	G3–M3	Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10
-Links to:	G4–M3	Multi-Digit Multiplication and Division
	G4–M7	Exploring Multiplication

In Lesson 1, students come to understand area as an attribute of plane figures that is defined by the amount of two-dimensional space within a bounded region. Students use pattern blocks to tile given polygons without gaps or overlaps, and without exceeding the boundaries of the shape.

Lesson 2 takes students into an exploration in which they cut apart paper rectangles into same-sized squares to concretely define a square unit, specifically square inches and centimeters. They use these units to make rectangular arrays that have the same area, but different side lengths.

Lessons 3 and 4 introduce students to the strategy of finding area using centimeter and inch tiles. Students use tiles to determine the area of a rectangle by tiling the region without gaps or overlaps. They then bring the ruler (with corresponding units) alongside the array to discover that the side length is equal to the number of tiles required to cover one side of the rectangle. From this experience, students begin to relate total area with multiplication of side lengths.

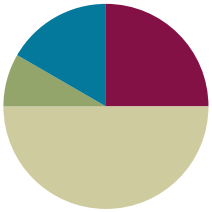
A Teaching Sequence Towards Mastery of Foundations for Understanding Area	
Objective 1:	**Understand area as an attribute of plane figures. (Lesson 1)**
Objective 2:	**Decompose and recompose shapes to compare areas. (Lesson 2)**
Objective 3:	**Model tiling with centimeter and inch unit squares as a strategy to measure area. (Lesson 3)**
Objective 4:	**Relate side lengths with the number of tiles on a side. (Lesson 4)**

Lesson 1

Objective: Understand area as an attribute of plane figures.

Suggested Lesson Structure

Fluency Practice	(15 minutes)
Application Problem	(5 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (15 minutes)

- Group Counting **3.OA.1** (4 minutes)
- Identify the Shape **2.G.1** (3 minutes)
- Find the Common Products **3.OA.7** (8 minutes)

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition.

Direct students to count forward and backward, occasionally changing the direction of the count.

- Threes to 30
- Sixes to 60
- Sevens to 70
- Eights to 80
- Nines to 90

Identify the Shape (3 minutes)

Materials: (T) Images of polygons (S) Personal white boards

Note: This fluency reviews properties and vocabulary that will be used during today's Concept Development.

T: (Project a triangle.) How many sides does this shape have?
S: 3.
T: Name the shape.
S: Triangle.

Continue with the following possible sequence: quadrilateral (trapezoid), quadrilateral (rhombus), quadrilateral (square), and quadrilateral (rectangle).

Find the Common Products (8 minutes)

Materials: (S) Blank paper

Note: This fluency reviews multiplication patterns from G3–Module 3.

T: Fold your paper in half vertically.

T: On the left half, count by twos to 20 down the side of your paper.

T: On the right half, count by fours to 40 down the side of your paper.

T: Draw lines to match multiples that appear in both columns.

S: (Match 4, 8, 12, 16, and 20.)

T: (Write ___ × 2 = 4, ___ × 2 = 8, etc., next to each corresponding product on the left half of the paper.) Write the complete equations next to their products.

S: (Write equations and complete unknowns.)

T: (Write 4 = ____ × 4, 8 = ____ × 4, etc., next to each corresponding product on the right half of the paper.) Write the complete equations next to their products.

S: (Write equations.)

T: (Write 2 × 2 = ____ × 4.) Say the equation including all factors.

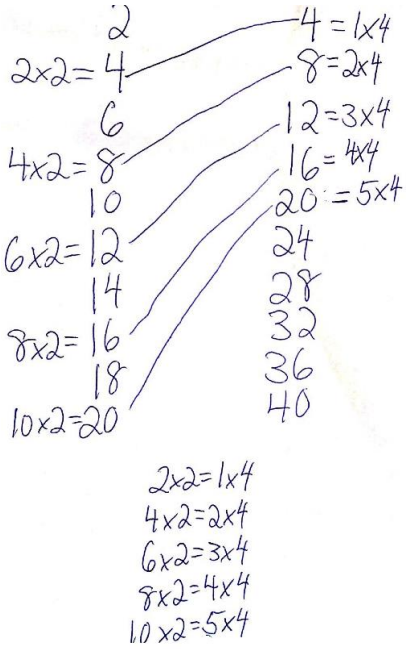
S: 2 × 2 = 1 × 4.

T: (Write 2 × 2 = 1 × 4.) Write the remaining equal facts as equations.

S: (Write 4 × 2 = 2 × 4, 6 × 2 = 3 × 4, 8 × 2 = 4 × 4, and 10 × 2 = 5 × 4.)

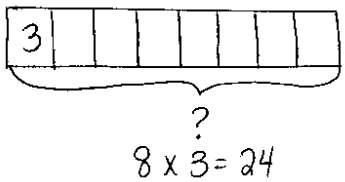
T: What patterns do you notice in your equations?

S: Each multiple of 4 is also a multiple of 2.



Application Problem (5 minutes)

Eric makes a shape with 8 trapezoid pattern blocks. Brock makes the same shape using triangle pattern blocks. It takes 3 triangles to make 1 trapezoid. How many triangle pattern blocks does Brock use?



Brock uses 24 triangle pattern blocks.

Note: This problem reviews the G3–Module 3 concept of multiplying using units of 8.

Concept Development (30 minutes)

Materials: (S) Pattern blocks, Problem Set

Part 1: Using pattern blocks to understand area.

T: Look at Problem 1 on your Problem Set. Discuss with a partner whether you think Shape A or Shape B takes up more space. Be prepared to explain your answer. (After students discuss, facilitate a whole class discussion.)

S: Shape A, because it's longer than Shape B. → Shape B, because it's taller than Shape A.

T: Use green triangle pattern blocks to cover Shape A and Shape B. Be sure the triangles do not have gaps between them, they don't overlap, and they don't go outside the sides of the shapes. (Allow time for students to work.) What did you notice about the number of green triangles it takes to cover Shape A and Shape B?

S: It takes 6 green triangles to cover each shape!

T: Answer Problem 1 on your Problem Set. (Allow time for students to write answers.) Do all the green triangles take up the same amount of space?

S: Yes, because they're all the same size.

T: What does that mean about the amount of space Shape A and Shape B take up?

S: They're the same. → It took 6 triangles to cover each shape, which means the shapes take up the same amount of space. → The amount of space that Shape A takes up is equal to the amount of space Shape B takes up.

T: The amount of flat space a shape takes up is called its **area**. Since Shapes A and B take up the same amount of space, their areas are equal.

Repeat the process of using pattern blocks to cover Shapes A and B with the blue rhombus and the red trapezoid pattern blocks. Students record their work on Problems 2 and 3 in the Problem Set.

T: What is the relationship between the size of the pattern blocks and the number of pattern blocks it takes to cover Shapes A and B?

S: The bigger the pattern block, the smaller the number of pattern blocks it takes to cover these shapes. → The bigger pattern blocks, like the trapezoid, cover more area than the triangles. That means it takes fewer trapezoids to cover the same area as the triangles.

T: Answer Problem 4 on your Problem Set.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Manipulating pattern blocks may be a challenge for some learners. Try the following tips:

- Partner students so they can work together to cover the shapes.
- Encourage students to hold the pattern blocks in place with one hand, while they place the remaining blocks.
- Instead of using pattern blocks, provide paper shapes that can be glued, so they won't move around unnecessarily.
- Offer the computer as a resource to create and move shapes.

Part 2: Measuring area using square units.

T: Use orange square pattern blocks to cover the rectangle in Problem 5. Be sure the squares don't have gaps between them, they don't overlap, and they don't go outside the sides of the rectangle. (Allow students time to work.) How many squares did it take to cover the rectangle?

S: 6!

T: Answer Problem 5 on your Problem Set. (Allow time for students to write answers.) The area of Shape C is 6 square units. Why do you think we call them square units?

S: Because they're squares! → The units used to measure are squares, so they're square units!

T: Yes! The units used to measure the area of the rectangle are squares.

T: Use red trapezoid pattern blocks to cover the rectangle in Problem 5. Be sure the trapezoids don't have gaps between them, they don't overlap, and they don't go outside the sides of the rectangle. (Allow students time to work.) What did you notice?

S: It's not possible! → The red trapezoids can't cover this shape without having gaps.

T: Use this information to help you answer Problem 6 on your Problem Set. (Allow time for students to write answers.) I'm going to say an area in square units, and you're going to make a rectangle with your pattern blocks that has that area. Which pattern blocks will you use?

S: The squares because the units for area that you're telling us are square units!

T: Here we go! Four square units.

S: (Make rectangles.)

Continue with the following possible suggestions: 12 square units, 9 square units, and 8 square units. Invite students to compare their rectangles to a partner's rectangles. How are they the same? How are they different? If time allows, students can work with a partner to create rectangles that have the same areas, but look different.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Students working above grade level can be encouraged to find other square units in the classroom that they can either use to make rectangles or that already form rectangles. Such items might include sticky notes, desktops, floor tiles, and linking cubes. Students can create a poster to share with the class that shows the areas of the rectangles made with these other square units.

Student Debrief (10 minutes)

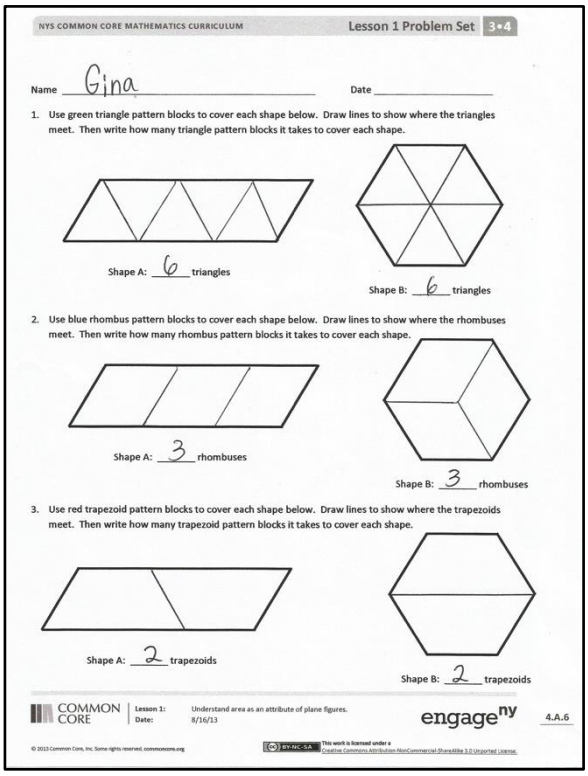
Lesson Objective: Understand area as an attribute of plane figures.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- Talk to a partner. Do you think you can use orange square pattern blocks to cover Shapes A and B in Problem 1? Explain your answer.
- How many green triangle pattern blocks does it take to cover a blue rhombus pattern block? Use that information to say a division fact that relates the number of triangles it takes to cover Shape A to the number of rhombuses it takes to cover the same shape. (6 ÷ 2 = 3.)
- Explain to a partner how you used orange square pattern blocks to find the area of the rectangle in Problem 5.
- What new math vocabulary did we use today to communicate precisely about the amount of space a shape takes up? (**Area**.) Which units did we use to measure area?
- How did the Application Problem connect to today's lesson?


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 1 Problem Set 3•4

Name Gina Date

1. Use green triangle pattern blocks to cover each shape below. Draw lines to show where the triangles meet. Then write how many triangle pattern blocks it takes to cover each shape.

Shape A: 6 triangles

Shape B: 6 triangles

2. Use blue rhombus pattern blocks to cover each shape below. Draw lines to show where the rhombuses meet. Then write how many rhombus pattern blocks it takes to cover each shape.

Shape A: 3 rhombuses

Shape B: 3 rhombuses

3. Use red trapezoid pattern blocks to cover each shape below. Draw lines to show where the trapezoids meet. Then write how many trapezoid pattern blocks it takes to cover each shape.

Shape A: 2 trapezoids

Shape B: 2 trapezoids

COMMON CORE Lesson 1: Understand area as an attribute of plane figures. Date: 8/16/13 engage^ny 4.A.6

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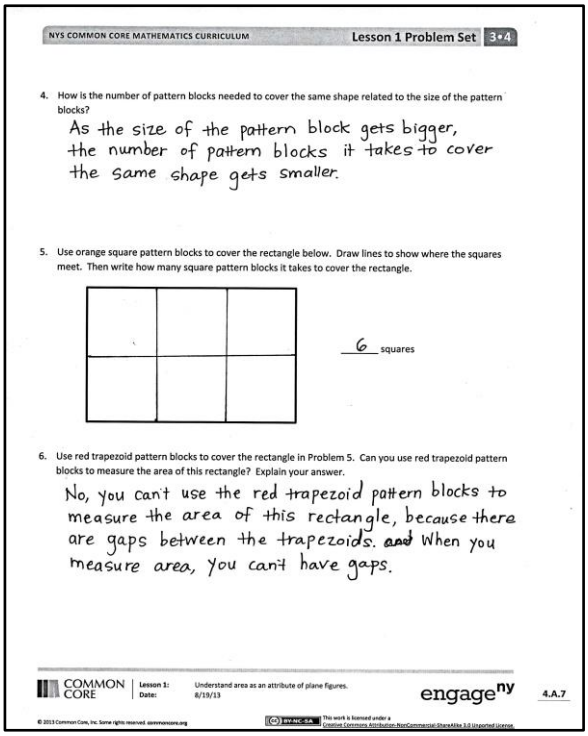
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4. How is the number of pattern blocks needed to cover the same shape related to the size of the pattern blocks?

As the size of the pattern block gets bigger, the number of pattern blocks it takes to cover the same shape gets smaller.

5. Use orange square pattern blocks to cover the rectangle below. Draw lines to show where the squares meet. Then write how many square pattern blocks it takes to cover the rectangle.

6 squares

6. Use red trapezoid pattern blocks to cover the rectangle in Problem 5. Can you use red trapezoid pattern blocks to measure the area of this rectangle? Explain your answer.

No, you can't use the red trapezoid pattern blocks to measure the area of this rectangle, because there are gaps between the trapezoids. ~~and~~ When you measure area, you can't have gaps.

COMMON CORE Lesson 1: Understand area as an attribute of plane figures. Date: 8/19/13 engage^ny 4.A.7

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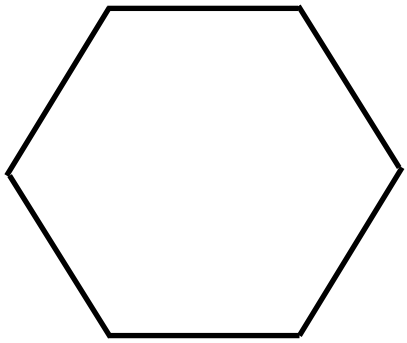
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

Name _____ Date _____

1. Use green triangle pattern blocks to cover each shape below. Draw lines to show where the triangles meet. Then write how many triangle pattern blocks it takes to cover each shape.

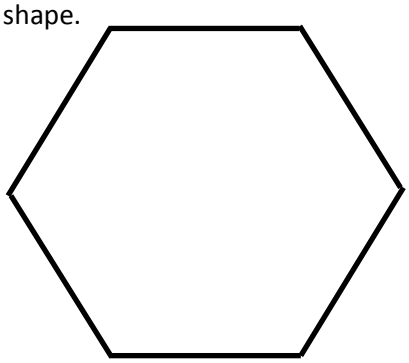
Shape A: _____ triangles



Shape B: _____ triangles

2. Use blue rhombus pattern blocks to cover each shape below. Draw lines to show where the rhombuses meet. Then write how many rhombus pattern blocks it takes to cover each shape.

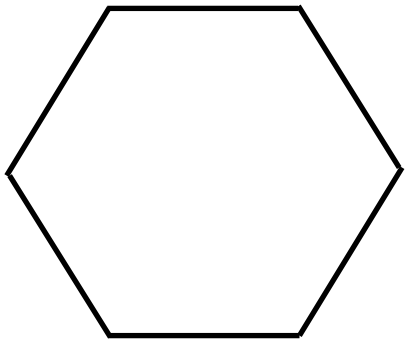
Shape A: _____ rhombuses



Shape B: _____ rhombuses

3. Use red trapezoid pattern blocks to cover each shape below. Draw lines to show where the trapezoids meet. Then write how many trapezoid pattern blocks it takes to cover each shape.

Shape A: _____ trapezoids



Shape B: _____ trapezoids

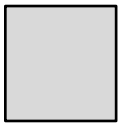
4. How is the number of pattern blocks needed to cover the same shape related to the size of the pattern blocks?

5. Use orange square pattern blocks to cover the rectangle below. Draw lines to show where the squares meet. Then write how many square pattern blocks it takes to cover the rectangle.

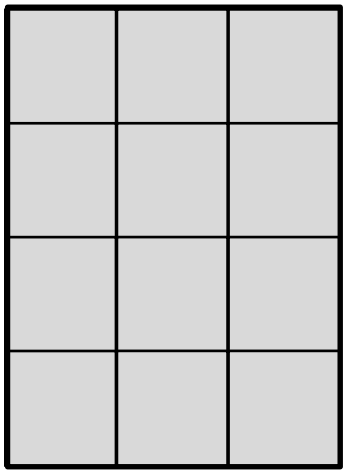
_____ squares

6. Use red trapezoid pattern blocks to cover the rectangle in Problem 5. Can you use red trapezoid pattern blocks to measure the area of this rectangle? Explain your answer.

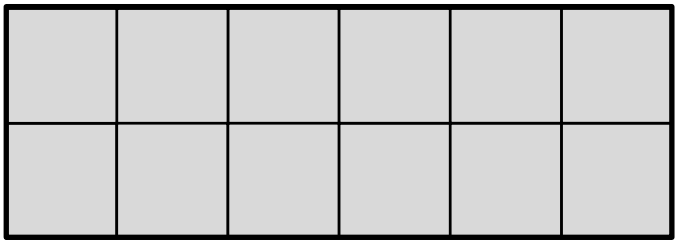
Name _____ Date _____

1. Each  is 1 square unit. Do both rectangles have the same area? Explain how you know.

a.



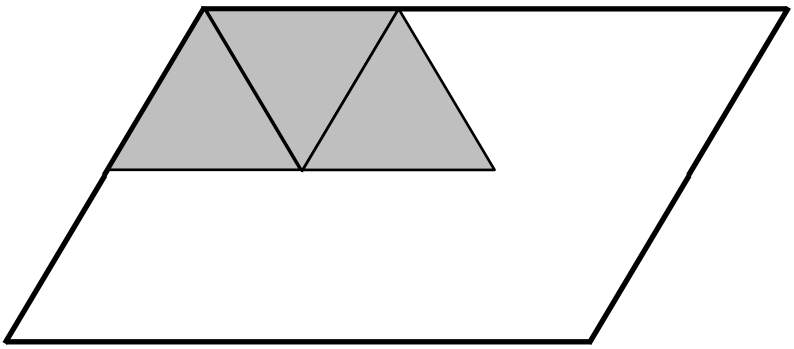
b.



Name _____ Date _____

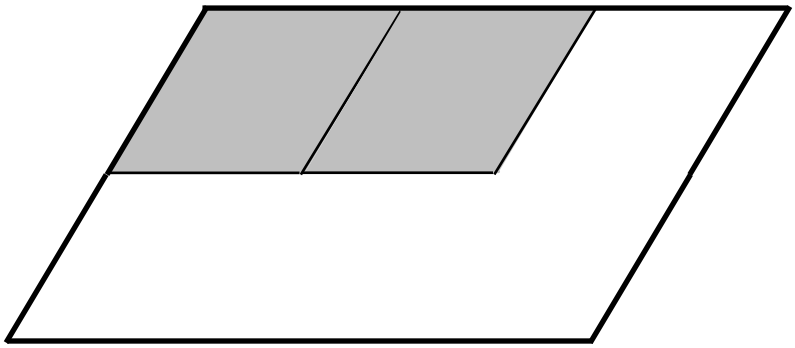
1. Magnus covers the same shape with triangles, rhombuses, and trapezoids

   a. How many triangles will it take to cover the shape?



_____ triangles

   b. How many rhombuses will it take to cover the shape?

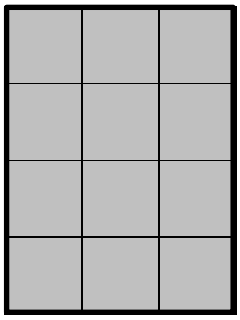


_____ rhombuses

   c. Magnus notices that 3 triangles from Part (a) cover 1 trapezoid. How many trapezoids will it take to cover the shape below? Explain your answer.

_____ trapezoids

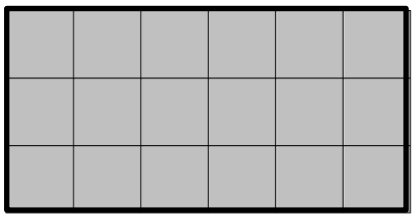
2. Angela uses squares to find the area of a rectangle. Her work is shown below.
   a. How many squares did she use to cover the rectangle?



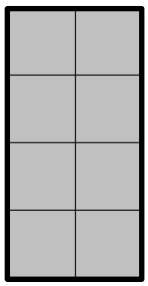
_____ squares

   b. What is the area of the rectangle in square units? Explain how you found your answer.

3. Each  is 1 square unit. Which rectangle has the biggest area? How do you know?



Rectangle A



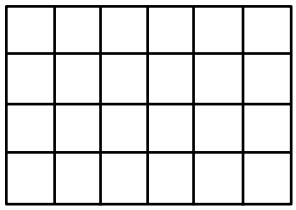
Rectangle B

Rectangle C

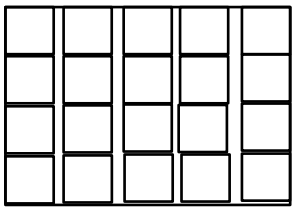
Name _____ Date _____

1. Jasmine and Roland each use unit squares to tile a piece of paper. Their work is shown below.

Jasmine's Array



Roland's Array



a. Can one of the arrays be used to correctly measure the area of the piece of paper? If so, whose array would you use? Explain why.

b. What is the area of the piece of paper? Explain your strategy for finding the area.

c. Jasmine thinks she can skip-count by sixes to find the area of her rectangle. Is she correct? Explain why or why not.

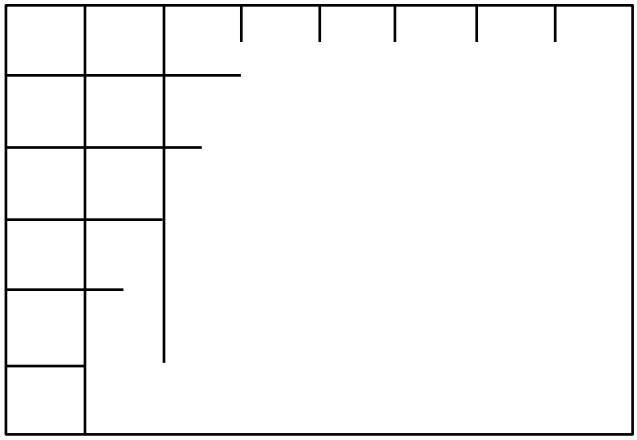
2. Jaheim says you can create three rectangles with different side lengths using 12 unit squares. Use numbers, equations, and words to show what Jaheim is saying.

3. The area of a rectangle is 72 square units. One side has a length of 9 units. What is the other side length? Explain how you know using pictures, equations, and words.

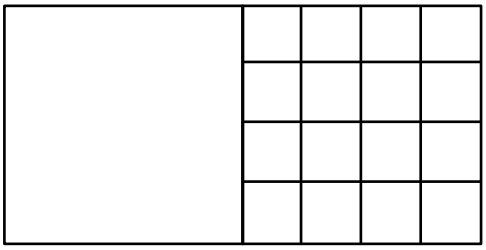
4. Jax started to draw a grid inside the rectangle to find its area.

   a. Use a straight edge to complete the drawing of the grid.

   b. Write both an addition and a multiplication equation that you could use to find the area, then solve.



5. Half of the rectangle below has been tiled with unit squares.



   a. How many more unit squares are needed to fill in the rest of the rectangle?

   b. What is the total area of the large rectangle? Explain how you found the area.

Mid-Module Assessment Task Standards Addressed	Topics A–B

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

3.MD.7 Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Evaluating Student Learning Outcomes

A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop *on their way to proficiency*. In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for each student is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the student CAN do now and what they need to work on next.

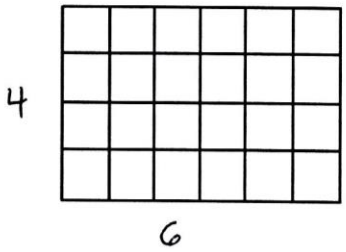
A Progression Toward Mastery				
Assessment Task Item and Standards Assessed	**STEP 1 Little evidence of reasoning without a correct answer. (1 Point)**	**STEP 2 Evidence of some reasoning without a correct answer. (2 Points)**	**STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)**	**STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)**
1 **3.MD.5** **3.MD.6**	Response demonstrates little evidence of reasoning without a correct answer.	Response shows limited reasoning with at least one correct answer.	Response includes evidence of some reasoning with three correct answers, or evidence of solid reasoning with an incorrect answer.	Student correctly answers: a. Jasmine's array, giving strong evidence of understanding that tiling must have no gaps or overlaps. b. The area is 24 tiles. Provides appropriate explanation of the calculation including counting or skip-counting strategies. c. Yes, there are 4 rows of 6 squares so it is possible to skip-count by six.
2 **3.MD.7b**	Response demonstrates little evidence of reasoning without a correct answer.	Response shows limited reasoning with at least one correct answer.	Student identifies at least two of three rectangles correctly. Response includes evidence of accurate reasoning with pictures, numbers, or words.	Student correctly identifies three rectangles: ▪ 1 × 12 or 12 × 1 ▪ 2 × 6 or 6 × 2 ▪ 3 × 4 or 4 × 3 Response shows evidence of solid reasoning using pictures, numbers, and words.

A Progression Toward Mastery				
3 **3.MD.7b**	Response demonstrates little evidence of reasoning without a correct answer.	Response shows limited reasoning without a correct answer.	Student finds the missing side length of 8 units but may not show enough work to clearly justify the answer.	Student correctly finds the missing side length of 8 units. Response shows evidence of solid reasoning using pictures, numbers, and words.
4 **3.MD.5** **3.MD.6** **3.MD.7a**	Response demonstrates little evidence of reasoning without a correct answer.	Response shows evidence of some reasoning in attempt to write equations and complete the array, but work may not include a correct answer.	Student accurately completes the array and finds the area of 48 sq cm, but may not accurately provide both addition and multiplication equations.	Student correctly: a. Completes the array with 8 columns and 6 rows. b. Writes an addition equation (repeated addition of 8 sixes or 6 eights); writes a multiplication equation (6 × 8 or 8 × 6); and gives an area of 48 sq cm.
5 **3.MD.5a** **3.MD.5b** **3.MD.7a** **3.MD.7d**	Response demonstrates little evidence of reasoning without a correct answer to either part.	Response shows limited reasoning with a correct answer in one part.	Student slightly miscalculates the number of tiles needed to fill the remaining area, but the explanation shows evidence of solid reasoning. Part (b) is correct based on the student's slight miscalculation but not the correct answer of 32 units.	Student correctly: a. Identifies that 16 tiles are needed to fill the remaining area. b. Says the area of the large rectangle is 32 square units. Explanation gives evidence of solid reasoning to support answer.

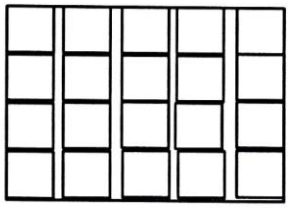
Name Gina Date

1. Jasmine and Roland each use unit squares to tile a piece of paper. Their work is shown below.

Jasmine's Array



Roland's Array



a. Can one of the arrays be used to correctly measure the area of the piece of paper? If so, whose array would you use? Explain why.

Jasmine's array correctly measures the area of the piece of paper. You can't have gaps or overlaps when you tile or it won't be right.

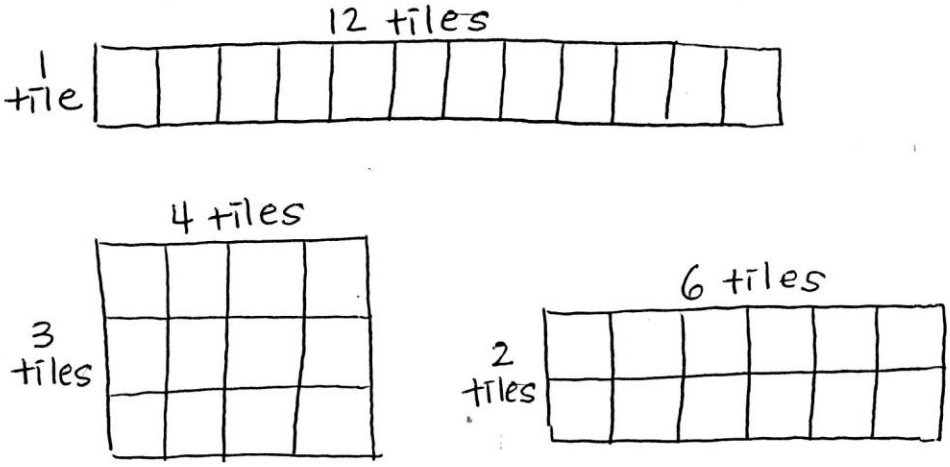
b. What is the area of the piece of paper? Explain your strategy for finding the area.

6 tiles across and 4 tiles down.
I can multiply 6 x 4 = 24 to find the area.
Roland's array is wrong so I have to use Jasmine's array.

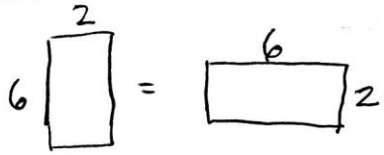
c. Jasmine thinks she can skip-count by sixes to find the area of her rectangle. Is she correct? Explain why or why not.

Yes, Jasmine is correct. There are 4 rows of six squares so she can skip-count: 6, 12, 18, 24.
It's faster if she multiplies.

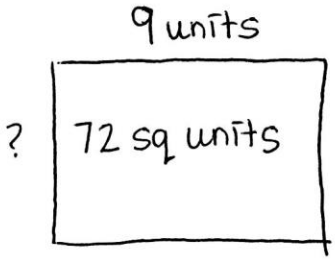
2. Jaheim says you can create three rectangles with different side lengths using 12 unit squares. Use numbers, equations, and words to show what Jaheim is saying.



Jaheim is correct. These are the only rectangles you can make with 12 tiles. You can turn them, but they will still be the same:



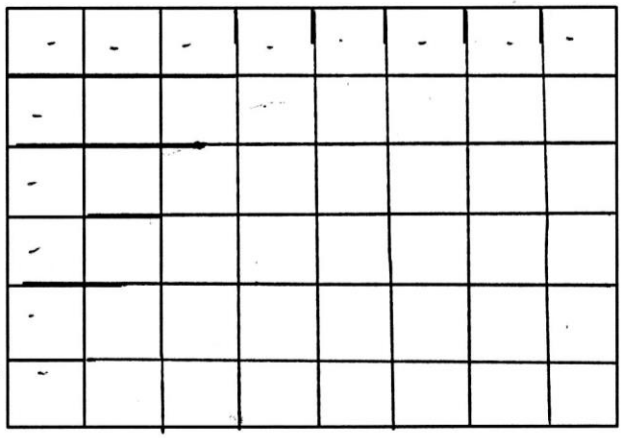
3. The area of a rectangle is 72 square units. One side has a length of 9 units. What is the other side length? Explain how you know using pictures, equations, and words.



$? \times 9 = 72$
$72 \div 9 = 8$

If one side length is 9 units, the other side length is 8 units because $8 \times 9 = 72$.

4. Jax started to draw a grid inside the rectangle to find its area.



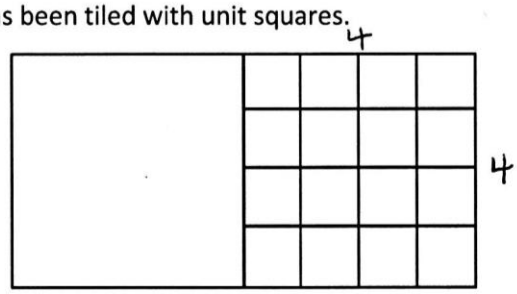
a. Use a straight edge to complete the drawing of the grid.

b. Write both an addition and a multiplication equation that you could use to find the area, then solve.

8 + 8 + 8 + 8 + 8 + 8 = 48 sq units

6 x 8 = 48 sq units

5. Half of the rectangle below has been tiled with unit squares.



4 x 4 = 16 sq units

a. How many more unit squares are needed to fill in the rest of the rectangle?

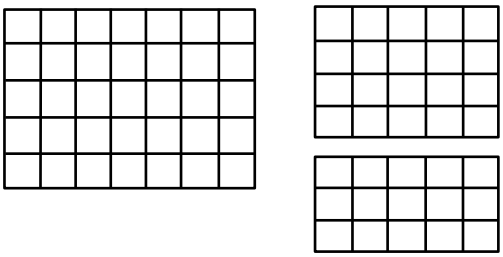
If there's 16 sq units in one half, there will be 16 sq units in the other half too. You need 16 more tiles to fill it in.

b. What is the total area of the large rectangle? Explain how you found the area.

16 + 16 = 32 sq units.
I added the 2 halves together to find the total area of the rectangle.

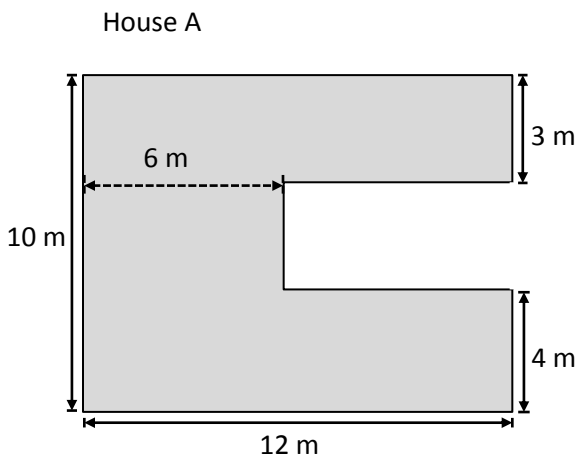
Name _____ Date _____

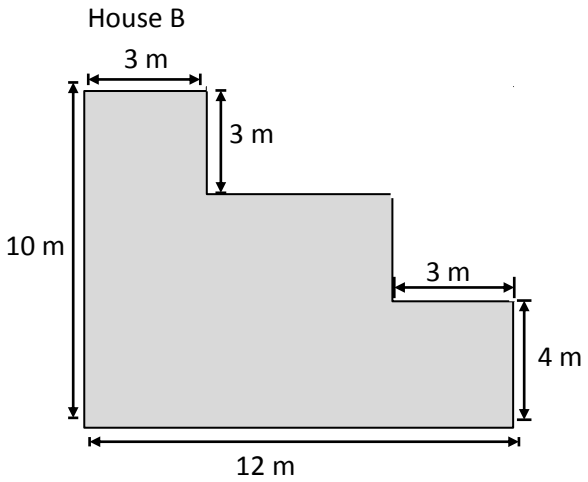
1. Sarah says the rectangle on the left has the same area as the sum of the two on the right. Pam says they do not have the same areas. Who is correct? Explain using numbers, pictures, or words.



2. Draw three different arrays that you could make with 36 square-inch tiles. Label the side lengths on each of your arrays. Write multiplication sentences for each array to prove that the area of each array is 36 square inches.

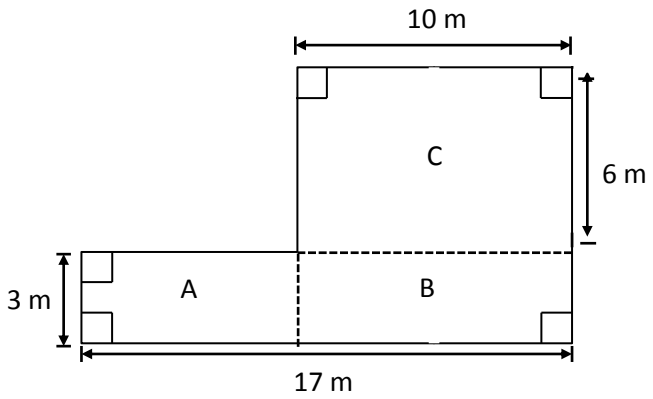
3. Mr. and Mrs. Jackson are buying a new house. They are deciding between the two floor plans below.





Which floor plan has the greater area? Show how you found your answer on the drawings above. Show your calculations below.

4. Superior Elementary School uses the design below for their swimming pool.



a. Label the side lengths of Rectangles A and B on the drawing.

b. Find the area of each rectangle.

c. Find the area of the entire pool. Explain how you found the area of the pool.

End-of-Module Assessment Task Standards Addressed	Topics A–D

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

3.MD.7 Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Evaluating Student Learning Outcomes

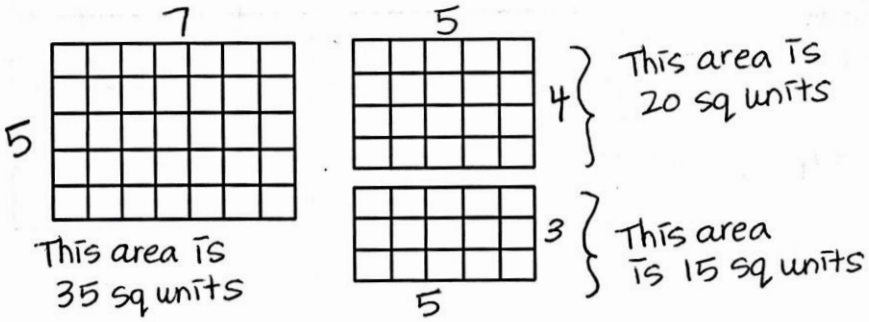
A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop *on their way to proficiency*. In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for each student is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the student CAN do now and what they need to work on next.

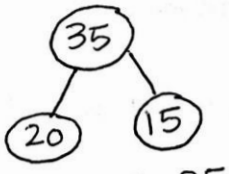
A Progression Toward Mastery				
Assessment Task Item and Standards Assessed	**STEP 1 Little evidence of reasoning without a correct answer.** (1 Point)	**STEP 2 Evidence of some reasoning without a correct answer or with a partially correct answer in a multi-step question.** (2 Points)	**STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer.** (3 Points)	**STEP 4 Evidence of solid reasoning with a correct answer.** (4 Points)
1 **3.MD.7c** **3.MD.7d**	Response demonstrates little or no evidence of reasoning without a correct answer.	Student identifies that Sarah is correct, demonstrating evidence of limited reasoning to support the answer.	Student identifies that Sarah is correct. Response shows evidence of accurate reasoning to support the answer using at least one representation.	Student identifies that Sarah is correct. Explanation shows evidence of solid reasoning using multiple representations.
2 **3.MD.5b** **3.MD.6** **3.MD.7a** **3.MD.7b**	Student attempts, but is unable to draw any correct arrays with labels. Multiplication sentences are not shown.	Student correctly draws and labels one array. Side lengths are labeled without units. A multiplication sentence is shown.	Student correctly draws and labels two different arrays. Side lengths are labeled in inches. Multiplication sentences are shown for those two arrays.	Student correctly draws and labels three different arrays. Side lengths are labeled in inches. Possible arrays: ▪ 1 × 36 ▪ 2 × 18 ▪ 3 × 12 ▪ 4 × 9 ▪ 6 × 6 Correct multiplication sentences are shown for each array drawn.
3 **3.MD.7d** **3.MD.7b**	Response demonstrates little or no evidence of reasoning without a correct answer.	Student miscalculates one area. Student may identify that House A has the greater area with limited reasoning.	Response demonstrates correct calculations and area. Student identifies that House A has the greater area.	Student demonstrates correct area calculations with answers: ▪ House A = 102 sq meters ▪ House B = 84 sq meters Explanation identifies that House A has the greater area. Response provides evidence of solid reasoning.

A Progression Toward Mastery				
4 **3.MD.5** **3.MD.7b** **3.MD.7d**	Attempts, but is unable to answer any part of the question correctly.	Student: a. Labels length and width correctly, but without units. b. Calculates at least two areas correctly. c. May miscalculate the total area.	Student answers Parts (a) and (b) correctly, but may miscalculate the total area.	Student correctly: a. Labels length and width of rectangles A and B, including units: ▪ A = 3 m × 7 m ▪ B = 3 m × 10 m b. Calculates the area of each rectangle as: ▪ A = 21 sq meters ▪ B = 30 sq meters ▪ C = 60 sq meters c. Calculates the total area as 111 sq meters.

Name Gina Date

1. Sarah says the rectangle on the left has the same area as the sum of the two on the right. Pam says they do not have the same areas. Who is correct? Explain using numbers, pictures, or words.

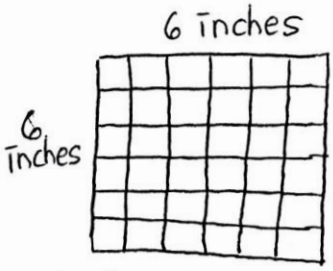




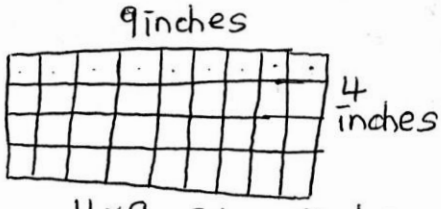
20+15=35

Sarah is right. The two on the right add up to 35 sq units, which is the area of the one on the left.

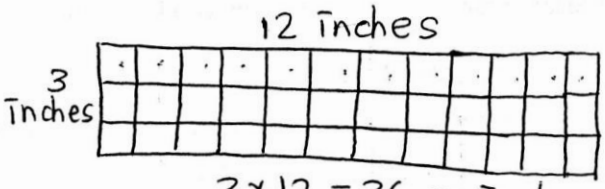
2. Draw three different arrays that you could make with 36 square-inch tiles. Label the side lengths on each of your arrays. Write multiplication sentences for each array to prove that the area of each array is 36 square inches.



6x6=36 sq inches

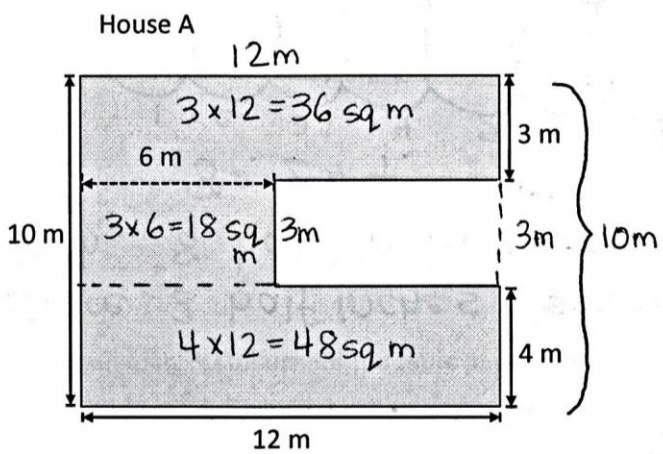


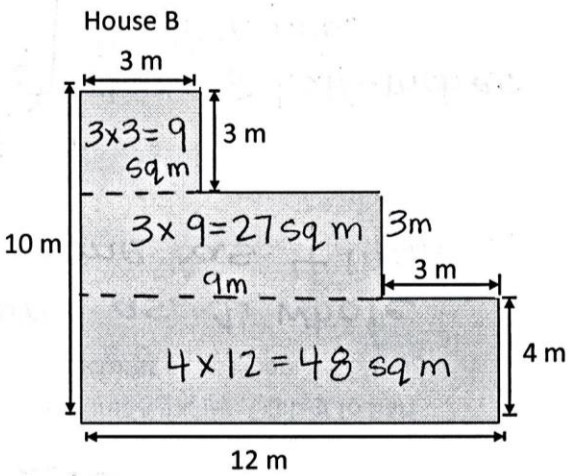
4x9=36 sq inches



3x12=36 sq inches

3. Mr. and Mrs. Jackson are buying a new house. They are deciding between the two floor plans below.





Which floor plan has the greater area? Show how you found your answer on the drawings above. Show your calculations below.

House A:

36 + 18 + 48

40 + 14

54 + 48

52 + 50

= 102 square meters

House B:

9 + 27 + 48

6 + 30

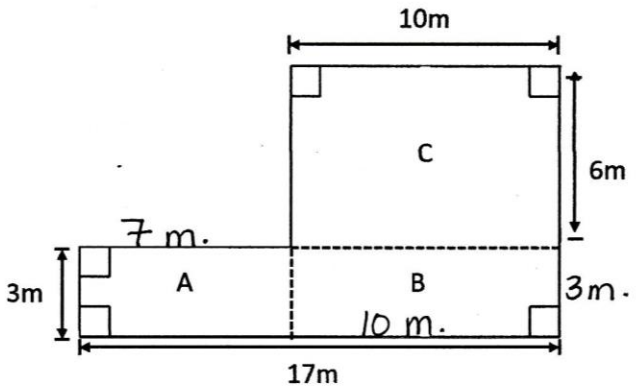
36 + 48

34 + 50

= 84 square meters

Mr. and Mrs. Jackson should buy House A, because it has a greater area than House B. House A is 102 square meters and House B is only 84 square meters.

4. Superior Elementary School uses the design below for their swimming pool.



a. Label the length and width of rectangles A and B on the drawing.

b. Find the area of each rectangle.

A → 7x3 = 21 square meters

B → 10x3 = 30 square meters

C → 10x6 = 60 square meters

c. Find the area of the entire pool. Explain how you found the area of the pool.

I can add the areas of all 3 parts to find the area of the whole pool.

21+30+60

21 + 90

111 square meters