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Relationships Between Quantities and Reasoning with Equations and Their Graphs

[1] Each lesson is ONE day, and ONE day is considered a 45 minute period.

Algebra I • Module 1

Relationships Between Quantities and Reasoning with Equations and Their Graphs

OVERVIEW

By the end of Grade 8, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students are introduced to non-linear equations and their graphs. They formalize their understanding of equivalent algebraic expressions and begin their study of polynomial expressions. Further, they learn that there are some actions that, when applied to the expressions on both sides of an equal sign, will not result in an equation with the same solution set as the original equation. Finally, they encounter problems that induce the full modeling cycle, as it is described in the Common Core Learning Standards for Mathematics.

In Topic A, students explore the main functions that they will work with in Grade 9: linear, quadratic, and exponential. The goal is to introduce students to these functions by having them make graphs of situations (usually based upon time) in which the functions naturally arise (**A-CED.2**). As they graph, they reason abstractly and quantitatively as they choose and interpret units to solve problems related to the graphs they create (**N-Q.1**, **N-Q.2**, **N-Q.3**).

In middle school, students applied the properties of operations to add, subtract, factor, and expand expressions (**6.EE.3**, **6.EE.4**, **7.EE.1**, **8.EE.1**). Now, in Topic B, students use the structure of expressions to define what it means for two algebraic expressions to be equivalent. In doing so, they discern that the commutative, associative, and distributive properties help link each of the expressions in the collection together, even if the expressions look very different themselves (**A-SSE.2**). They learn the definition of a polynomial expression and build fluency in identifying and generating polynomial expressions as well as adding, subtracting, and multiplying polynomial expressions (**A-APR.1**). The Mid-Module Assessment follows Topic B.

Throughout middle school, students practice the process of solving linear equations (**6.EE.5**, **6.EE.7**, **7.EE.4**, **8.EE.7**) and systems of linear equations (**8.EE.8)**. Now, in Topic C, instead of just solving equations, they formalize descriptions of what they learned before (variable, solution sets, etc.) and are able to explain, justify, and evaluate their reasoning as they strategize methods for solving linear and non-linear equations (**A-REI.1**, **A-REI.3**, **A-CED.4**). Students take their experience solving systems of linear equations further as they prove the validity of the addition method, learn a formal definition for the graph of an equation and use it to explain the reasoning of solving systems graphically, and graphically represent the solution to systems of linear inequalities (**A-CED.3**, **A-REI.5**, **A-REI.6**, **A-REI.10**, **A-REI.12**).

In Topic D, students are formally introduced to the modeling cycle (see page 61 of the CCLS) through problems that can be solved by creating equations and inequalities in one variable, systems of equations, and graphing (**N-Q.1**, **A-SSE.1**, **A-CED.1**, **A-CED.2**, **A-REI.3**). The End-of-Module Assessment follows Topic D.

Focus Standards

Reason quantitatively and use units to solve problems.

N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★

N-Q.2[2] Define appropriate quantities for the purpose of descriptive modeling.★

N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

Interpret the structure of expressions

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

A-SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Perform arithmetic operations on polynomials

A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Create equations that describe numbers or relationships

A-CED.1[3] Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

[2] This standard will be assessed in Algebra I by ensuring that some modeling tasks (involving Algebra I content or securely held content from Grades 6-8) require the student to create a quantity of interest in the situation being described.
[3] In Algebra I, tasks are limited to linear, quadratic, or exponential equations with integer exponents.

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★

A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*★

Understand solving equations as a process of reasoning and explain the reasoning

A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve systems of equations

A-REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.6[4] Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically

A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Foundational Standards

Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.7 Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*

[4] Tasks have a real-world context. In Algebra I, tasks have hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.*

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities.

6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or depending on the purpose at hand, any number in a specified set.

6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

6.EE.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Use properties of operations to generate equivalent expressions.

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Work with radicals and integer exponents.

8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.7 Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.8 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.*

c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students are presented with problems that require them to try special cases and simpler forms of the original problem in order to gain insight into the problem.

MP.2 **Reason abstractly and quantitatively.** Students analyze graphs of non-constant rate measurements and reason from the shape of the graphs to infer what quantities are being displayed and consider possible units to represent those quantities.

MP.3 **Construct viable arguments and critique the reasoning of others.** Students reason about solving equations using "if-then" moves based on equivalent expressions and properties of equality and inequality. They analyze when an "if-then" move is not reversible.

MP.4 **Model with mathematics.** Students have numerous opportunities in this module to solve problems arising in everyday life, society, and the workplace from modeling bacteria growth to understanding the federal progressive income tax system.

MP.6 **Attend to precision.** Students formalize descriptions of what they learned before (variables, solution sets, numerical expressions, algebraic expressions, etc.) as they build equivalent expressions and solve equations. Students analyze solution sets of equations to determine processes (like squaring both sides of an equation) that might lead to a solution set that differs from that of the original equation.

MP.7 **Look for and make use of structure.** Students reason with and about collections of equivalent expressions to see how all the expressions in the collection are linked together through the properties of operations. They discern patterns in sequences of solving equation problems that reveal structures in the equations themselves: $2x + 4 = 10$, $2(x - 3) + 4 = 10$, $2(3x - 4) + 4 = 10$, etc.

MP.8 **Look for and express regularity in repeated reasoning.** After solving many linear equations in one variable (e.g., $3x + 5 = 8x - 17$), students look for general methods for solving a generic linear equation in one variable by replacing the numbers with letters: $ax + b = cx + d$. They have opportunities to pay close attention to calculations involving the properties of operations, properties of equality, and properties of inequality as they find equivalent expressions and solve equations, noting common ways to solve different types of equations.

Terminology

New or Recently Introduced Terms

- **Piecewise-Linear Function** (Given a finite number of non-overlapping intervals on the real number line, a *(real) piecewise-linear function* is a function from the union of the intervals to the set of real numbers such that the function is defined by (possibly different) linear functions on each interval.)
- **Numerical Symbol** (A *numerical symbol* is a symbol that represents a specific number.)
- **Variable Symbol** (A variable symbol is a symbol that is a placeholder for a number. It is possible that a question may restrict the type of number that a placeholder might permit, maybe integers only or a positive real number, for instance.)
- **Numerical Expression** (A *numerical expression* is an algebraic expression that contains only numerical symbols (no variable symbols) and that evaluates to a single number.)
- **Algebraic Expression** (An *algebraic expression* is either (1) a numerical symbol or a variable symbol or (2) the result of placing previously generated algebraic expressions into the two blanks of one of the four operators ((__)+(__), (__)−(__), (__)×(__), (__)÷(__)) or into the base blank of an exponentiation with an exponent that is a rational number.)
- **Equivalent Numerical Expressions** (Two numerical expressions are *equivalent* if they evaluate to the same number.)
- **Equivalent Algebraic Expressions** (Two algebraic expressions are *equivalent* if we can convert one expression into the other by repeatedly applying the Commutative, Associative, and Distributive Properties and the properties of rational exponents to components of the first expression.)
- **Polynomial Expression** (A *polynomial expression* is either (1) a numerical expression or a variable symbol or (2) the result of placing two previously generated polynomial expressions into the blanks of the addition operator (__+__) or the multiplication operator (__×__).)
- **Monomial** (A *monomial* is a polynomial expression generated using only the multiplication operator (__×__). Monomials are products whose factors are numerical expressions or variable symbols.)
- **Degree of a Monomial** (The *degree* of a non-zero monomial is the sum of the exponents of the variable symbols that appear in the monomial.)
- **Standard Form of a Polynomial Expression in One Variable** (A polynomial expression with one variable symbol x is in *standard form* if it is expressed as $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$, where n is a non-negative integer, and $a_0, a_1, a_2, \ldots, a_n$ are constant coefficients with $a_n \neq 0$. A polynomial expression in x that is in standard form is often called a *polynomial in* x.)
- **Degree of a Polynomial in Standard Form** (The *degree of a polynomial in standard form* is the highest degree of the terms in the polynomial, namely n.)
- **Leading Term and Leading Coefficient of a Polynomial in Standard Form** (The term a_nx^n is called the *leading term,* and a_n is called the *leading coefficient.*)
- **Constant Term of a Polynomial in Standard Form** (The *constant term* is the value of the numerical expression found by substituting 0 into all the variable symbols of the polynomial, namely a_0.)
- **Solution** (A *solution* to an equation with one variable is a number in the domain of the variable that, when substituted for all instances of the variable in both expressions, makes the equation a true number sentence.)

- **Solution Set** (The set of solutions of an equation is called its *solution set.*)
- **Graph of an Equation in Two Variables** (The set of all points in the coordinate plane that are solutions to an equation in two variables is called the *graph of the equation.*)
- **Zero Product Property** (The Zero Product Property states that given real numbers, a and b, if $a \cdot b = 0$ then either $a = 0$ or $b = 0$, or both a and $b = 0$.)

Familiar Terms and Symbols[5]

- Equation
- Identity
- Inequality
- System of Equations
- Properties of Equality
- Properties of Inequality
- Solve
- Linear Function
- Formula
- Term

Suggested Tools and Representations

- Coordinate Plane
- Equations and Inequalities

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	N-Q.1, N-Q.2, N-Q.3, A-APR.1, A-SSE.2
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	N-Q.1, A-SSE.1, A-SSE.2, A-APR.1, A-CED.1, A-CED.2, A-CED.3, A-CED.4, A-REI.1, A-REI.3, A-REI.5, A-REI.6, A-REI.10, A-REI.12

[5] These are terms and symbols students have seen previously.

Topic A:

Introduction to Functions Studied This Year—Graphing Stories

N-Q.1, N-Q.2, N-Q.3, A-CED.2

Focus Standard:	N-Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; and choose and interpret the scale and the origin in graphs and data displays.
	N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.
	N-Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
	A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
Instructional Days:	5	
Lesson 1:	Graphs of Piecewise Linear Functions	
Lesson 2:	Graphs of Quadratic Functions	
Lesson 3:	Graphs of Exponential Functions	
Lesson 4:	Analyzing Graphs—Water Usage During a Typical Day at School	
Lesson 5:	Two Graphing Stories	

Students explore the main functions that they will work with in Grade 9: linear, quadratic, and exponential. The goal is to introduce students to these functions by having them make graphs of a situation (usually based upon time) in which these functions naturally arise. As they graph, they reason quantitatively and use units to solve problems related to the graphs they create.

For example, in Lesson 3 they watch a 20-second video that shows bacteria subdividing every few seconds. The narrator of the video states these bacteria are actually subdividing every 20 minutes. After counting the initial number of bacteria and analyzing the video, students are asked to create the graph to describe the number of bacteria with respect to actual time (not the sped-up time in the video) and to use the graph to approximate the number of bacteria shown at the end of the video.

Another example of quantitative reasoning occurs in Lesson 4. Students are shown a graph (without labels) of the water usage rate of a high school. The rate jumps every hour for five minutes and then drops back down, supposedly during the bell breaks between classes. As students interpret the graph, they are asked to choose and interpret the scale and decide on the level of accuracy of the measurements needed to capture the behavior in the graph.

The topic ends with a lesson that introduces the next two topics on expressions and equations. Students are asked to graph two stories that intersect in one point on the same coordinate plane. After students and teachers form linear equations to represent both graphs and use those equations to find the intersection point (a Grade 8 standard, **8.EE.8**), the question is posed to students: How can we use algebra in general to solve problems like this one but for non-linear equations? Topics B and C set the stage for students' understanding of the general procedure for solving equations.

Lesson 1: Graphs of Piecewise Linear Functions

Student Outcomes

- Students define appropriate quantities from a situation (a "graphing story"), choose and interpret the scale and the origin for the graph, and graph the piecewise linear function described in the video. They understand the relationship between physical measurements and their representation on a graph.

Classwork

Example 1 (20 minutes)

Show the first $1:08$ minutes of video below, telling the class that our goal will simply be to describe the motion of the man in words. (Note: Be sure to stop the video at 1:08 because after that the answers to the graphing questions are given.)

Elevation vs. Time #2 [http://www.mrmeyer.com/graphingstories1/graphingstories2.mov. This is the second video under "Download Options" at the site http://blog.mrmeyer.com/?p=213 called "Elevation vs. Time #2."]

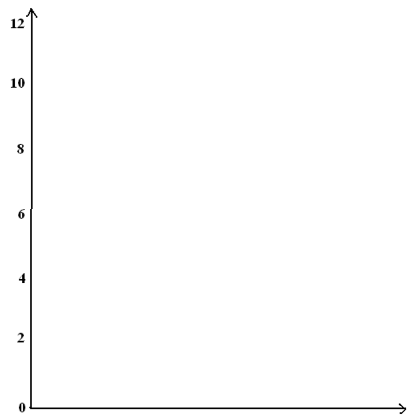
MP.1 After viewing the video, have students share out loud their ideas on describing the motion. Some might speak in terms of speed, distance traveled over time, or change of elevation. All approaches are valid. Help students begin to shape their ideas with precise language.

Direct the class to focus on the change of elevation of the man over time and begin to put into words specific details linking elevation with time.

- "How high do you think he was at the top of the stairs? How did you estimate that elevation?"
- "Were there intervals of time when his elevation wasn't changing? Was he still moving?"
- "Did his elevation ever increase? When?"

Help students discern statements relevant to the chosen variable of elevation.

If students do not naturally do so, suggest representing this information on a graph. As per the discussion that follows, display a set of axes on the board with vertical axis labeled in units relevant to the elevation.



Ask these types of questions:

- "How should we label the vertical axis? What unit of measurement should we choose (feet or meters)?"
- "How should we label the horizontal axis? What unit of measurement should we choose?"
- "Should we measure the man's elevation to his feet or to his head on the graph?"
- "The man starts at the top of the stairs. Where would that be located on the graph?"
- "Show me with your hand what the general shape of the graph should look like."

MP.6 Give time for students to draw the graph of the story (alone or in pairs). Lead a discussion through the issues of formalizing the diagram: The labels and units of the axes, a title for the graph, the meaning of a point plotted on the graph, a method for finding points to plot on the graph, and so on.

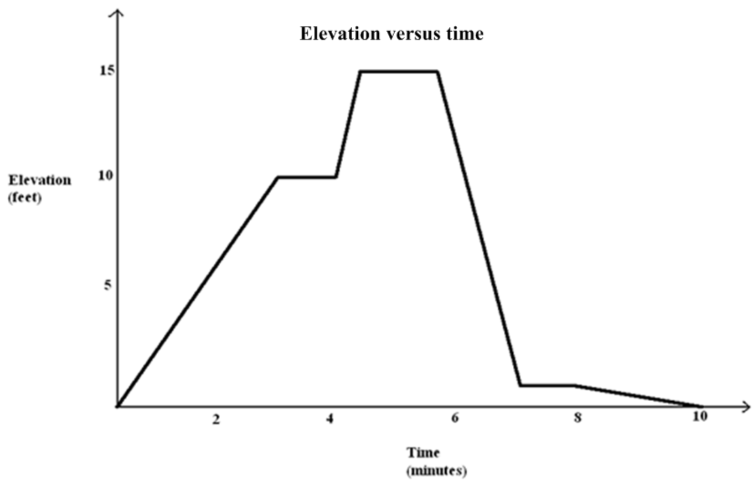
MP.3 NOTE: The graph shown at the end of the video is incorrect! The man starts at "30 feet above the ground," which is clearly false. You might ask students, "Can you find the error made in the video?"

Example 2 (15 minutes)

Present the following graph and question

Example 2

Here is an elevation-versus-time graph of a person's motion. Can we describe what the person might have been doing?



Have students discuss this question in pairs or in small groups. It will take some imagination to create a context that matches the shape of the graph, and there will likely be debate.

Additional questions to ask:

- What is happening in the story when the graph is increasing, decreasing, constant over time?
  - *Answers will vary depending on story: person is "walking up a hill," etc.*
- What does it mean for one part of the graph to be steeper than another?
  - *The person is climbing or descending faster than in the other part.*
- How does slope of each line segment relate to the context of the person's elevation?
  - *The slope gives the average change in elevation per minute.*
- Is it reasonable that a person moving up and down a vertical ladder could have produced this elevation versus time graph?
  - *It is unlikely because the speed is too slow:* 2.5 *feet per minute. If the same graph had units in seconds then it would be reasonable.*
- Is it possible for someone walking on a hill to produce this elevation versus time graph AND return to her starting point at the 10-minute mark? If it is, describe what the hill might look like.
  - *Yes, the hill could have a long path with a gentle slope that would zigzag back up to the top and then a shorter, slightly steeper path back down to the beginning position.*

- What was the average rate of change of the person's elevation between time 0 minutes and time 4 minutes?
    - $\frac{10}{4}\ ft./min\ or\ 2.5\ ft./min.$

These types of questions help students understand that the graph represents only elevation, not speed nor horizontal distance from the starting point. This is an important observation.

Closing (5 minutes)

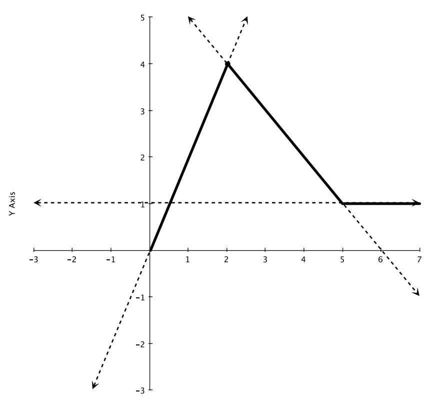
Ask the following:

- How would you describe the graph of Example 2 to a friend?
- What type of equation(s) would be required to create this graph?

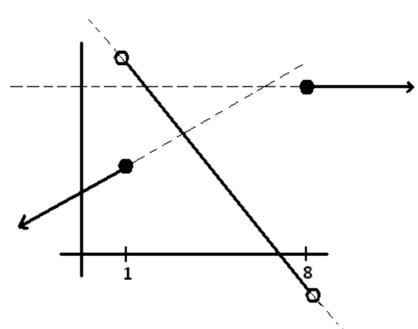
Introduce the following definition to your students and discuss briefly. (We will return to this definition later in the year.)

<u>PIECEWISE-DEFINED LINEAR FUNCTION</u>: Given non-overlapping intervals on the real number line, a *(real) piecewise linear function* is a function from the union of the intervals on the real number line that is defined by (possibly different) linear functions on each interval.

Point out that all graphs we studied today are graphs of piecewise linear functions. Remind students (see Standard 8.F.3) that the graphs of linear functions are straight lines and show how each segment in one of the graphs studied today is part of a straight line as in:



or



Also show students the intervals on which each linear function is defined. One may wish to point out there might be ambiguity as to whether or not the endpoints of a given interval belong to that interval. For example, in the first diagram we could argue that three linear functions are defined on the intervals $[0,2)$, $[2,5)$, and $[5,\infty)$, or perhaps on the intervals $[0,2]$, $(2,5)$, and $[5,\infty)$ instead. (Warning: Your students have not been formally introduced to interval notation.) There is no ambiguity in the second example. This point about the interval endpoints is subtle and is not an issue to focus on in a concerted way in this particular lesson.

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 1: Graphs of Piecewise Linear Functions

Exit Ticket

The graph in Example 1 is made by combining pieces of nine linear functions (it is a piecewise linear function). Each linear function is defined over an interval of time, represented on the horizontal axis. List those nine time intervals.

Exit Ticket Sample Solutions

Students may describe the intervals in words. Do not worry about the endpoints of the intervals in this lesson.

The graph in Example 1 is made by combining pieces of nine linear functions (it is a piecewise linear function). Each linear function is defined over an interval of time, represented on the horizontal axis. List those nine time intervals.

Between 0 and 3 seconds;

Between 3 and 5.5 seconds;

Between 5.5 and 7 seconds;

Between 7 and 8.5 seconds;

Between 8.5 and 9 seconds;

Between 9 and 11 seconds;

Between 11 and 12.7 seconds;

Between 12.7 and 13 seconds;

And 13 seconds onwards.

Problem Set Sample Solutions

1. **Watch the video, "Elevation vs. Time #3" (below)**

   http://www.mrmeyer.com/graphingstories1/graphingstories3.mov. (This is the third video under "Download Options" at the site http://blog.mrmeyer.com/?p=213 called "Elevation vs. Time #3.")

   It shows a man climbing down a ladder that is 10 feet high. At time 0 seconds, his shoes are at 10 feet above the floor, and at time 6 seconds, his shoes are at 3 feet. From time 6 seconds to the 8.5 second mark, he drinks some water on the step 3 feet off the ground. Afterward drinking the water, he takes 1.5 seconds to descend to the ground and then he walks into the kitchen. The video ends at the 15 second mark.

   a. **Draw your own graph for this graphing story. Use straight line segments in your graph to model the elevation of the man over different time intervals. Label your x-axis and y-axis appropriately and give a title for your graph.**

      [See video for one example of a graph of this story.]

   b. **Your picture is an example of a graph of a piecewise linear function. Each linear function is defined over an interval of time, represented on the horizontal axis. List those time intervals.**

      The intervals are $[0, 6]$, $(6, 8.5]$, $(8.5, 10]$, and $(10, 15]$, with the understanding that the inclusions of the endpoints may vary. Students may use any notation they want to describe the intervals.

   c. **In your graph in part (a), what does a horizontal line segment represent in the graphing story?**

      It is a period of time when he is neither going up or down.

   d. **If you measured from the top of the man's head instead (he is 6.2 feet tall), how would your graph change?**

      The whole graph would be shifted up 6.2 feet.

e. **Suppose the ladder is descending into the basement of the apartment. The top of the ladder is at ground level (0 feet) and the base at the ladder is 10 feet below ground level. How would your graph change in observing the man following the same motion descending the ladder?**

The whole graph would be shifted downwards 10 feet.

f. **What is his average rate of descent between time 0 seconds and time 6 seconds? What was his average rate of descent between time 8.5 seconds and time 10 seconds? Over which interval does he descend faster? Describe how your graph in part a can also be used to find the interval during which he is descending fastest.**

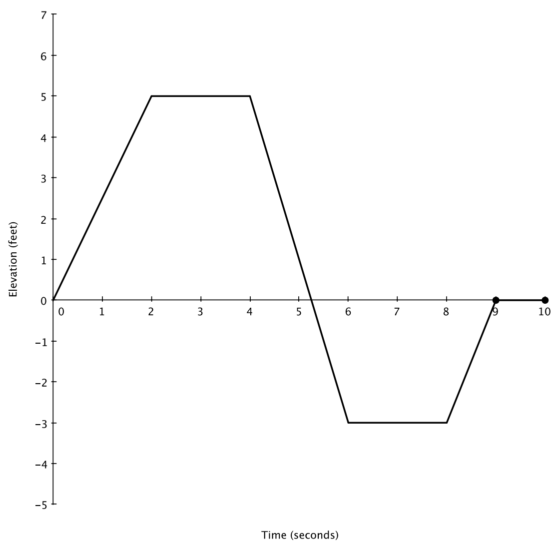
His average rate of descent between 0 and 6 seconds was $\frac{7}{6}$ ft/sec.

His average rate of descent between 8.5 and 10 seconds was 2 ft/sec.

He was descending faster from 8.5 to 10 seconds.

The interval during which he is descending the fastest corresponds to the line segment with the steepest negative slope.

2. **Make up an elevation-versus-time graphing story for the following graph:**



Answers will vary. A story along such as the following fits the graph:

A swimmer climbs a ladder to a waterslide, sits for two seconds at the top of the slide, and then slides down the slide into water. She stayed steady at the same position underwater for two seconds before rising to the surface.

Teachers should also accept other contexts, e.g., interpreting "0 elevation" as the height of a deck 3 feet above ground.

3. **Draw up an elevation-versus-time graphing story of your own and then make up a story for it.**

Answers will vary. Do not be too critical of their graphs and stories.

Name ____ Date ______________________

1. Jacob lives on a street that runs east and west. The grocery store is to the east and the post office is to the west of his house. Both are on the same street as his house. Answer the questions below about the following story:

   At 1:00 p.m., Jacob hops in his car and drives at a constant speed of 25 mph for 6 minutes to the post office. After 10 minutes at the post office, he realizes he is late and drives at a constant speed of 30 mph to the grocery store, arriving at 1:28 p.m. He then spends 20 minutes buying groceries.

   a. Draw a graph that shows the distance Jacob's car is from his house with respect to time. Remember to label your axes with the units you chose and any important points (home, post office, grocery store).

b. On the way to the grocery store, Jacob looks down at his watch and notes the time as he passes his house. What time is it when he passes his house? Explain how you found your answer.

c. If he drives directly back to his house after the grocery story, what was the total distance he traveled to complete his errands? Show how you found your answer.

2. Jason is collecting data on the rate of water usage in the tallest skyscraper in the world during a typical day. The skyscraper contains both apartments and businesses. The electronic water meter for the building displays the total amount of water used in liters. At noon, Jason looks at the water meter and notes that the digit in the **ones** place on the water meter display changes too rapidly to read the digit and that the digit in the **tens** place changes every second or so.

   a. Estimate the total number of liters used in the building during one 24-hour day. Take into account the time of day when he made his observation. (Hint: Will water be used at the same rate at 2:00 a.m. as at noon?). Explain how you arrived at your estimate.

   b. To what level of accuracy can Jason reasonably report a measurement if he takes it at precisely 12:00 p.m.? Explain your answer.

   c. The meter will be checked at regular time spans (for example, every minute, every 10 minutes, every hour). What is the minimum (or smallest) number of checks needed in a 24-hour period to create a reasonably accurate graph of the water usage **rate** with respect to time? (For example, 24 checks would mean checking the meter every hour; 48 checks would mean checking the meter every half hour.) Defend your choice by describing how the water usage rate might change during the day and how your choice could capture that change.

3. A publishing company orders black and blue ink in bulk for its two-color printing press. To keep things simple with its ink supplier, each time it places an order for blue ink, it buys B gallons, and each time it places an order for black ink, it buys K gallons. Over a one-month period, the company places m orders of blue ink and n orders of black ink.

   a. What quantities could the following expressions represent in terms of the problem context?

   $m + n$

   $mB + nK$

   $$\frac{mB + nK}{m + n}$$

   b. The company placed twice as many orders for black ink than for blue ink in January. Give interpretations for the following expressions in terms of the orders placed in January,

   $$\frac{m}{m+n} \quad \text{and} \quad \frac{n}{m+n},$$

   and explain which expression must be greater using those interpretations.

4. Sam says that he knows a clever set of steps to rewrite the expression

$$(x+3)(3x+8)-3x(x+3)$$

as a sum of two terms where the steps do not involve multiplying the linear factors first and then collecting like terms. Rewrite the expression as a sum of two terms (where one term is a number and the other is a product of a coefficient and variable) using Sam's steps if you can.

5. Using only the addition and multiplication operations with the numbers 1, 2, 3, and 4 each exactly once, it is possible to build a numeric expression (with parentheses to show the order used to build the expression) that evaluates to 21. For example, $1+((2+3)\cdot 4)$ is one such expression.

   a. Build two more numeric expressions that evaluate to 21 using the criteria above. Both must be different from the example given.

b. In both of your expressions, replace 1 with a, 2 with b, 3 with c, and 4 with d to get two algebraic expressions. For example, $a + ((b + c) \cdot d)$ shows the replacements for the example given.

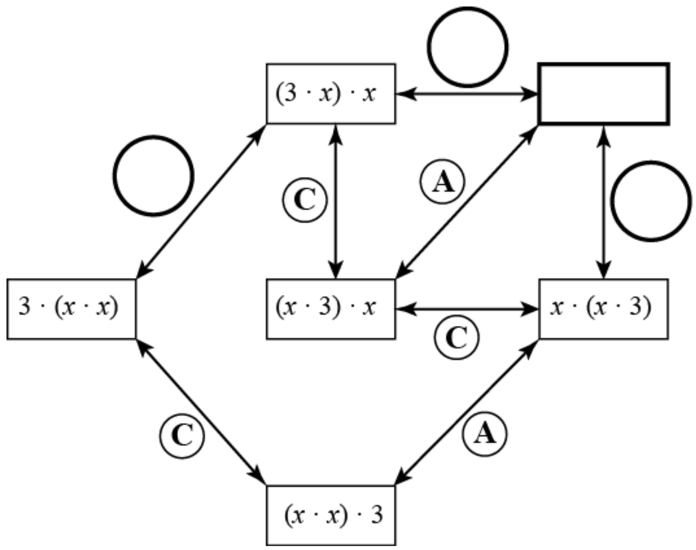
Are your algebraic expressions equivalent? Circle: Yes No

- If they are equivalent, prove that they are using the properties of operations.
- If not, provide **two** examples:

  (1) Find four different numbers (other than 0, 1, 2, 3, 4) that when substituted for *a, b, c,* and *d* into each expression, the expressions evaluate to **different numbers**, and

  (2) Find four different, non-zero numbers that when substituted into each expression, the expressions evaluate to the **same number**.

6. The diagram below, when completed, shows all possible ways to build equivalent expressions of $3x^2$ using multiplication. The equivalent expressions are connected by labeled segments stating which property of operations, **A** for Associative Property and **C** for Commutative Property, justifies why the two expressions are equivalent. Answer the following questions about $3x^2$ and the diagram.



   a. Fill in the empty circles with **A** or **C** and the empty rectangle with the missing expression to complete the diagram.
   b. Using the diagram above to help guide you, give *two different* proofs that $(x \cdot x) \cdot 3 = (3 \cdot x) \cdot x$.

7. Ahmed learned: "To multiply a whole number by ten, just place a zero at the end of the number." For example, 2813×10, he says, is 28,130. He doesn't understand why this "rule" is true.
   a. What is the product of the polynomial, $2x^3 + 8x^2 + x + 3$, times the polynomial, x?

   b. Use part (a) as a hint. Explain why the rule Ahmed learned is true.

8.

a. Find the following products:

i. $(x-1)(x+1)$

ii. $(x-1)(x^2 + x + 1)$

iii. $(x-1)(x^3 + x^2 + x + 1)$

iv. $(x-1)(x^4 + x^3 + x^2 + x + 1)$

v. $(x-1)(x^n + x^{n-1} + \cdots + x^3 + x^2 + x + 1)$

b. Substitute $x = 10$ into each of the products and your answers to show how each of the products appears as a statement in arithmetic.

c. If we substituted $x = 10$ into the product $(x-2)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ and computed the product, what number would result?

d. Multiply $(x-2)$ and $(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$, and express your answer in standard form.

Substitute $x=10$ into your answer, and see if you obtain the same result that you obtained in part (c).

e. Francois says $(x-9)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$ must equal $x^7+x^6+x^5+x^4+x^3+x^2+x+1$ because when $x=10$, multiplying by "$x-9$" is the same as multiplying by 1.

i. Multiply $(x-9)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$

ii. Put $x=10$ into your answer.

Is it the same as $x^7+x^6+x^5+x^4+x^3+x^2+x+1$ with $x = 10$?

iii. Was Francois right?

A Progression Toward Mastery					
Assessment Task Item		**STEP 1** **Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.**	**STEP 2** **Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.**	**STEP 3** **A correct answer with some evidence of reasoning or application of mathematics to solve the problem <u>or</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.**	**STEP 4** **A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.**
1	**a** **N-Q.1** **N-Q.2**	Student was unable to respond to question, <u>OR</u> student provided a minimal attempt to create an incorrect graph.	Graph reflects something related to the problem, but the axes do not depict the correct units of distance from the house on the y-axis and a measurement of time on the x-axis, <u>OR</u> the graph indicates significant errors in calculations or reasoning.	Student created axes that depict distance from the house on the y-axis and some measurement of time on the x-axis, and the graph represents a reflection of what occurred but with errors in calculations, missing or erroneous axis labels, <u>or</u> choice of units that makes the graph difficult to obtain information from.	Student created and labeled the y-axis to represent distance from the house in miles and an x-axis to represent time (in minutes past 1:00 p.m.) <u>AND</u> created a graph based on solid reasoning and correct calculations.
	b **N-Q.1**	Student answered incorrectly with no evidence of reasoning to support the answer, <u>OR</u> student left item blank.	Student answered incorrectly but demonstrated some reasoning in explaining the answer.	Student answered 1:21 p.m. but did not either refer to a correct graph or provide sound reasoning to support the answer. <u>OR</u> Student answered incorrectly because either the graph in part (a) was incorrect and the graph was	Student answered 1:21 p.m. <u>AND</u> either referred to a correct graph from part (a) or provided reasoning and calculations to explain the answer.

				referenced or because a minor calculation error was made but sound reasoning was used.	
	c **N-Q.1**	Student answered incorrectly with no evidence of reasoning to support the answer, <u>OR</u> student left item blank.	Student answered incorrectly but demonstrated some reasoning in explaining the answer.	Student answered 6 miles but did not either refer to the work in part (a) or provide sound reasoning in support of the answer. <u>OR</u> Student answered incorrectly because either the work in part (a) was referenced, but the work was incorrect or because a minor calculation error was made but sound reasoning was used.	Student answered 6 miles and either referenced correct work from part (a) or provided reasoning and calculations to support the answer.
2	**a** **N-Q.3**	Student left the question blank, <u>OR</u> student provided an answer that reflected no or very little reasoning.	Student either began with an assumption that was not based on the evidence of water being used at a rate of approximately 10 liters/second at noon, <u>OR</u> student used poor reasoning in extending that reading to consider total use across 24 hours.	Student answered beginning with the idea that water was being used at a rate of approximately 10 liters/second at noon but made an error in the calculations to extend and combine that rate to consider usage across 24 hours. <u>OR</u> Student did not defend the choice by explaining water usage across the 24 hours and how it compares to the reading taken at noon.	Student answered beginning with the idea that water was being used at a rate of approximately 10 liters/second at noon <u>AND</u> made correct calculations to extend and combine that rate to consider usage across 24 hours. <u>AND</u> Student defended the choice by explaining water usage across the 24 hours and how it compares to the reading taken at noon.

	b N-Q.3	Student left the question blank, OR student provided an answer that reflected no or very little reasoning.	Student answer is outside of the range from "to the nearest ten" to "to the nearest hundred", OR student answer is within that range but is not supported by an explanation.	Student answer ranges from "to the nearest ten" to "to the nearest hundred" but is not well supported by sound reasoning, OR student answer contains an error in the way the explanation was written, even if it was clear what the student meant to say.	Student answer ranges from "to the nearest ten" to "to the nearest hundred" AND is supported by correct reasoning that is expressed accurately.
	c N-Q.3	Student left the question blank, OR student provided an answer that reflected no or very little reasoning.	Student answer is not in the range of 6 to 48 checks but provides some reasoning to justify the choice, OR student answer is in that range, perhaps written in the form of 'every *x* minutes' or 'every *x* hours' but is not supported by an explanation with solid reasoning.	Student answer is in the range of 6 to 48 checks but is only given in the form of *x* checks per minute or *x* checks per hour; the answer is well supported by a written explanation, OR student answer is given in terms of number of checks but is not well supported by a written explanation.	Student answer is in the range of 6 to 48 checks, AND student provided solid reasoning to support the answer.
3	**a** A-SSE.1a A-SSE.1b	Student either did not answer, OR student answered incorrectly for all three expressions.	Student answered one or two of the three correctly but left the other one blank or made a gross error in describing what it represented.	Student answered two of the three correctly AND made a reasonable attempt at describing what the other one represented.	Student answered all three correctly.
	b A-SSE.1a A-SSE.1b	Student either did not answer, OR student answered incorrectly for all three parts of the question.	Student understood that the expressions represented a portion of the orders for each color but mis-assigned the colors and/or incorrectly determined which one would be larger.	Student understood that the expressions represented a portion of the orders for each color AND correctly determined which one would be larger but had errors in the way the answer was worded OR did not provide support for	Student understood that the expressions represented a portion of the orders for each color, correctly determined which one would be larger, AND provided a well written explanation for why.

				why $n/(m+n)$ would be larger.	
4	**A-SSE.1b** **A-SSE.2**	Student left the question blank, OR student was not able to re-write the expression successfully, even by multiplying out the factors first.	Student got to the correct re-written expression of 8(x + 3) but did so by multiplying out the factors first OR did not show the work needed to demonstrate how 8(x + 3) was determined.	Student attempted to use structure to re-write the expression as described, showing the process, but student made errors in the process.	Student correctly used the process described to arrive at 8(x + 3) without multiplying out linear factors AND demonstrated the steps for doing so.
5	**a – b** **A-SSE.2**	Student was unable to respond to many of the questions, OR student left several items blank.	Student was only able to come up with one option for part (a) and, therefore, had only partial work for part (b), OR student answered "Yes" for the question about equivalent expressions.	Student successfully answered part (a) AND identified that the expressions created in part (b) were not equivalent, but there were minor errors in the answering of the remaining questions.	Student answered all four parts correctly AND completely.
6	**a** **A-SSE.2**	Student left at least three items blank, OR student answered at least three items incorrectly.	Student answered one or two items incorrectly or left one or more items blank.	Student completed circling task correctly, AND provided a correct ordering of symbols in the box, but the answer did not use parentheses or multiplication dots.	Student completed all four item correctly, including exact placement of parentheses AND symbols for the box: $x \cdot (3 \cdot x)$.
	b **A-SSE.2**	Student did not complete either proof successfully.	Student attempted both proofs but made minor errors in both, OR student only completed one proof, with or without errors.	Student attempted both proofs but made an error in one of them.	Student completed both proofs correctly, AND the two proofs were different from one another.
7	**a** **A-APR.1**	Student's work is blank or demonstrates no understanding of multiplication of polynomials.	Student made more than one error in his multiplication but demonstrates some understanding of multiplication of polynomials.	Student made a minor error in the multiplication.	Student multiplied correctly and expressed the resulting polynomial as a sum of monomials.

	b A-APR.1	Student's explanation is missing or did not demonstrate a level of thinking that was higher than what was given in the problem's description of Ahmed's thinking.	Student used language that did not indicate an understanding of base x and/or the place value system. Student may have used language such as shifting or moving.	Student made only minor errors in the use of mathematically correct language to relate the new number to the old in terms of place value and/or the use of base x.	Student made no errors in the use of mathematically correct language to relate the new number to the old in terms of place value and/or the use of base x.
8	**a–c** A-APR.1	Student showed limited or no understanding of polynomial multiplication and of evaluating a polynomial for the given value of x.	Student made multiple errors but showed some understanding of polynomial multiplication. Student may not have combined like terms to present the product as the sum of monomials.	Student made one or two minor errors but demonstrated knowledge of polynomial multiplication and combining like terms to create the new polynomial. Student also showed understanding of evaluating a polynomial for the given value of x.	Student completed all products correctly, expressing each as a sum of monomials with like terms collected, and evaluated correctly when x is 10.
	d A-APR.1	Student showed limited or no understanding of polynomial multiplication and of evaluating a polynomial for the given value of x.	Student made multiple errors but showed some understanding of polynomial multiplication. Student may not have combined like terms to present the product as the sum of monomials. Student may have gotten an incorrect result when evaluating with $x = 10$.	Student made one or two minor errors but demonstrated knowledge of polynomial multiplication and combining like terms to create the new polynomial. Student also showed understanding of evaluating a polynomial for the given value of x.	Student correctly multiplied the polynomials and expressed the product as a polynomial in standard form. Student correctly evaluated with a value of 10 and answered "Yes".

	e **A-APR.1**	Student was not able to demonstrate an understanding that part iii is "No" and/or demonstrated limited or no understanding of polynomial multiplication.	Student may have some errors as he multiplied the polynomials and expressed the product as a sum of monomials. Student may have some errors in the calculation of the value of the polynomial when x is 10. Student incorrectly answered part iii or applied incorrect reasoning.	Student may have made minor errors in multiplying the polynomials and expressing the product as a sum of monomials. Student may have made minor errors in calculating the value of the polynomial when x is 10. Student explained that the hypothesized equation being true when $x = 10$ does not make it true for all real x and/or explained that the two expressions are not algebraically equivalent.	Student correctly multiplied the polynomials and expressed the product as a sum of monomials with like terms collected. Student correctly calculated the value of the polynomial when x is 10. Student explained that the hypothesized equation being true when $x = 10$ does not make it true for all real x and/or explained that the two expressions are not algebraically equivalent.

Name ______________________________ Date ____________________

1. Jacob lives on a street that runs east and west. The grocery store is to the east and the post office is to the west of his house. Both are on the same street as his house. Answer the questions below about the following story:

   At 1:00 p.m., Jacob hops in his car and drives at a constant speed of 25 mph for 6 minutes to the post office. After 10 minutes at the post office, he realizes he is late, and drives at a constant speed of 30 mph to the grocery store, arriving at 1:28 p.m. He then spends 20 minutes buying groceries.

   a. Draw a graph that shows the distance Jacob's car is from his house with respect to time. Remember to label your axes with the units you chose and any important points (home, post office, grocery store).

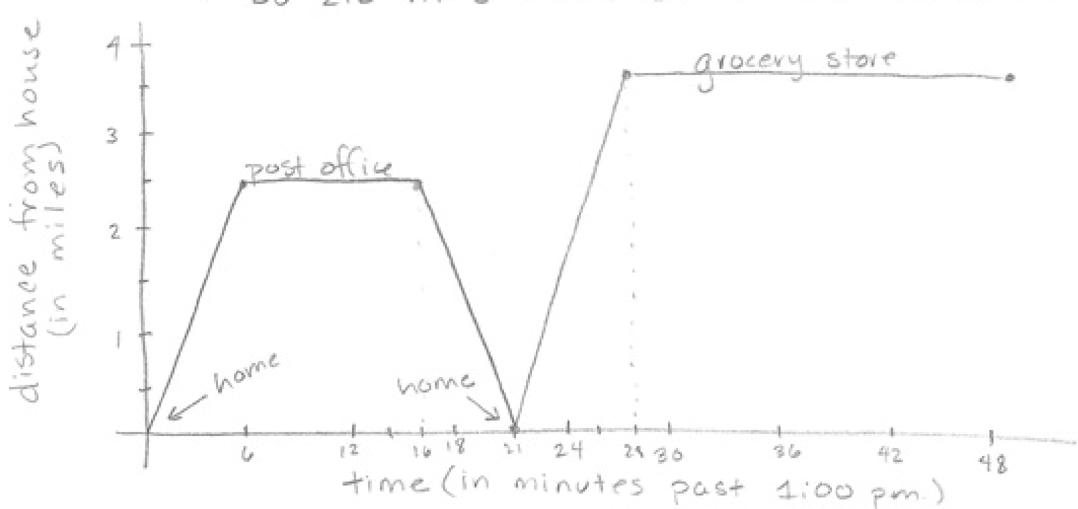
$25 \frac{\text{miles}}{\text{hour}} \times 6\text{min} \times \frac{1\text{hr}}{60\text{min}} =$ 2.5 miles from house to post office.

$30 \frac{\text{miles}}{\text{hour}} \times 12\text{ min} \times \frac{1\text{hr}}{60\text{min}} =$ 6 miles from post office to store.

$6\text{mi} - 2.5\text{mi} =$ 3.5 miles from home to store.

6 miles in 12 min is 1 mile in 2 min

So 2.5 miles takes 5 min and 3.5 miles takes 7 min.

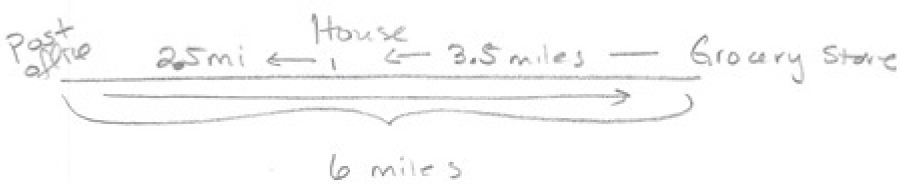


b. On the way to the grocery store, Jacob looks down at his watch and notes the time as he passes his house. What time is it when he passes his house? Explain how you found your answer.

It is 1:21. The graph shows the time as 21 minutes past 1:00 pm. He spent 6 minutes getting to the post office, 10 minutes at the post office and 5 min getting from the post office to the point of passing by his house. You know it took 5 minutes for the last part because he traveled 30 miles per hour & went 2.5 miles. $2.5\text{ mi} \cdot \frac{60\text{ min}}{30\text{ miles}} = 5\text{ min.}$

c. If he drives directly back to his house after the grocery story, what was the total distance he traveled to complete his errands? Show how you found your answer.

12 miles.



2.5 mi + 6 miles + 3.5 miles = 12 miles

You know that it is 2.5 miles from the House to the post office because $25\frac{\text{mi}}{\text{hr}} \times 6\text{min} \times \frac{1\text{hr}}{60\text{min}} = 2.5$ miles.

You know it is 6 miles from the post office to the store because $30\frac{\text{mi}}{\text{hr}} \times 12\text{min} \times \frac{1\text{hr}}{60\text{min}} = 6$ miles.

2. Jason is collecting data on the rate of water usage in the tallest skyscraper in the world during a typical day. The skyscraper contains both apartments and businesses. The electronic water meter for the building displays the total amount of water used in liters. At noon, Jason looks at the water meter and notes that the digit in the **ones** place on the water meter display changes too rapidly to read the digit and that the digit in the **tens** place changes every second or so.

   a. Estimate the total number of liters used in the building during one 24-hour day. Take into account the time of day when he made his observation. (Hint: Will water be used at the same rate at 2:00 a.m. as at noon?). Explain how you arrived at your estimate.

$$\begin{array}{r} \overset{4}{36} \\ \times 18 \\ \hline 288 \\ 360 \\ \hline 648 \end{array}$$

$$10\ \frac{\text{liters}}{\text{sec}} \times 60\ \frac{\text{sec}}{\text{min}} \times 60\ \frac{\text{min}}{\text{hr}} \times 18\ \text{hr} = 648{,}000\ \text{liters}.$$

Since water is probably only used from about 5:00 am to 11:00 pm, I did not multiply by 24 hours, but by 18 hours instead.

   b. To what level of accuracy can Jason reasonably report a measurement if he takes it at precisely 12:00 p.m.? Explain your answer.

It can be reported within ± 10 liters, since he can read the 10's place, but it is changing by a 10 during the second he reads it

   c. The meter will be checked at regular time spans (for example, every minute, every 10 minutes, every hour). What is the minimum (or smallest) number of checks needed in a 24-hour period to create a reasonably accurate graph of the water usage **rate** with respect to time? (For example, 24 checks would mean checking the meter every hour; 48 checks would mean checking the meter every half hour.) Defend your choice by describing how the water usage rate might change during the day and how your choice could capture that change.

24 checks.

Every hour would be good to show the peaks in usage during morning and evening hours from those in the apartments. And it might also show that businesses stop using it after business hours. It would depend on what portion of the building is business vs. apartments.

3. A publishing company orders black and blue ink in bulk for its two-color printing press. To keep things simple with its ink supplier, each time it places an order for blue ink, it buys B gallons, and each time it places an order for black ink, it buys K gallons. Over a one-month period, the company places m orders of blue ink and n orders of black ink.

   a. What quantities could the following expressions represent in terms of the problem context?

   $m + n$ total number of ink orders over a one month period.

   $mB + nK$ total gallons of ink ordered over a one month period.

   $\frac{mB + nK}{m + n}$ average number of gallons of ink per order.

   b. The company placed twice as many orders for black ink than for blue ink in January. Give interpretations for the following expressions in terms of the orders placed in January,

   $$\frac{m}{m+n} \quad \text{and} \quad \frac{n}{m+n},$$

   and explain which expression must be greater using those interpretations.

   $\frac{m}{m+n}$ is the fraction of orders that are for blue ink.

   $\frac{n}{m+n}$ is the fraction of orders that are for black ink.

   $\frac{n}{m+n}$ would be bigger, 2 times as big as $\frac{m}{m+n}$ because they ordered twice as many orders for black ink than blue ink.

4. Sam says that he knows a clever set of steps to rewrite the expression

$$(x+3)(3x+8)-3x(x+3)$$

as a sum of two terms where the steps do not involve multiplying the linear factors first and then collecting like terms. Rewrite the expression as a sum of two terms (where one term is a number and the other is a product of a coefficient and variable) using Sam's steps if you can.

$$((3x+8)-3x)(x+3)$$

$$8(x+3)$$

5. Using only the addition and multiplication operations with the numbers 1, 2, 3, and 4 each exactly once, it is possible to build a numeric expression (with parentheses to show the order used to build the expression) that evaluates to 21. For example, 1+((2+3) · 4) is one such expression.

   a. Build two more numeric expressions that evaluate to 21 using the criteria above. Both must be different from the example given.

$(1+2)\cdot(3+4) = 21$

$((2+4)+1)\cdot 3 = 21$

b. In both of your expressions, replace 1 with *a,* 2 with *b*, 3 with *c,* and 4 with *d* to get two algebraic expressions. For example, $a + ((b + c) \cdot d)$ shows the replacements for the example given.

$(a+b)\cdot(c+d) = ac+ad+bc+bd$

$((b+d)+a)\cdot c = ac + bc + dc$

Are your algebraic expressions equivalent? Circle: Yes No

- If they are equivalent, prove that they are using the properties of operations.
- If not, provide **two** examples:

(1) Find four different numbers (other than 0, 1, 2, 3, 4) that when substituted for *a, b, c,* and *d* into each expression, the expressions evaluate to **different numbers**, and

$a=5 \quad b=10 \quad c=20 \quad d=30$

$(5+10)\cdot(20+30) = 750$

$((10+30)+5)20 = 900$

(2) Find four different, non-zero numbers that when substituted into each expression, the expressions evaluate to the **same number**.

5, 6, 11, 7

$(5+6)(11+7) = 11\cdot18 = 198$

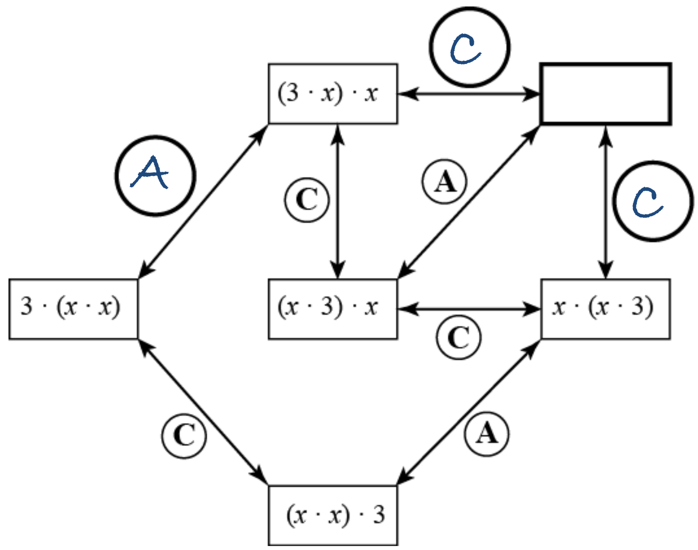
$((6+7)+5)\cdot11 = 18\cdot11 = 198$

$(ac+ad+bc+bd)$ needs to equal $(ac+bc+dc)$; so $(ad+bd)$ needs to equal (dc); So $(a+b)$ needs to equal c

$((3x+8)-3x)(x+3)$

$8(x+3)$

6. The diagram below, when completed, shows all possible ways to build equivalent expressions of $3x^2$ using multiplication. The equivalent expressions are connected by labeled segments stating which property of operations, **A** for Associative Property and **C** for Commutative Property, justifies why the two expressions are equivalent. Answer the following questions about $3x^2$ and the diagram.



a. Fill in the empty circles with **A** or **C** and the empty rectangle with the missing expression to complete the diagram.

b. Using the diagram above to help guide you, give *two different* proofs that $(x \cdot x) \cdot 3 = (3 \cdot x) \cdot x$.

① $(x \cdot x)3 = x(x \cdot 3)$ by Associative Property
$x(x \cdot 3) = x(3 \cdot x)$ by Commutative Property
$x(3 \cdot x) = (3 \cdot x) \cdot x$ by Commutative Property

② $(x \cdot x) \cdot 3 = 3 \cdot (x \cdot x)$ by Commutative Property
$3 \cdot (x \cdot x) = (3 \cdot x) \cdot x$ by Associative Property

7. Ahmed learned: "To multiply a whole number by ten, just place a zero at the end of the number." For example, 2813×10, he says, is 28,130. He doesn't understand why this "rule" is true.

a. What is the product of the polynomial, $2x^3 + 8x^2 + x + 3$, times the polynomial, x?

$2x^4 + 8x^3 + x^2 + 3x$

b. Use part (a) as a hint. Explain why the rule Ahmed learned is true.

When you multiply by the same number as the base, it creates a new number where each digit in the original number is now one place-value higher so that there is nothing left (no numbers) to represent the ones' digit, which leads to a trailing "0" in the ones' digit.

8.

a. Find the following products:

i. $(x-1)(x+1)$

$x^2 + x - x - 1$

$x^2 - 1$

ii. $(x-1)\left(x^2 + x + 1\right)$

$x^3 + x^2 + x - x^2 - x - 1$

$x^3 - 1$

iii. $(x-1)(x^3 + x^2 + x + 1)$

$x^4 + x^3 + x^2 + x - x^3 - x^2 - x - 1$

$x^4 - 1$

iv. $(x-1)(x^4 + x^3 + x^2 + x + 1)$

$x^5 + x^4 + x^3 + x^2 + x - x^4 - x^3 - x^2 - x - 1$

$x^5 - 1$

v. $(x-1)(x^n + x^{n-1} + \cdots + x^3 + x^2 + x + 1)$

$x^{n+1} - 1$

b. Substitute $x = 10$ into each of the products and your answers to show how each of the products appears as a statement in arithmetic.

i. $(10 - 1)(10 + 1) = (100 - 1)$

$9(11) = 99$

ii. $(10 - 1)(100 + 10 + 1) = (1000 - 1)$

$9(111) = 999$

iii. $(10 - 1)(1000 + 100 + 10 + 1) = (10{,}000 - 1)$

$9(1111) = 9999$

iv. $(10 - 1)(10{,}000 + 1000 + 100 + 10 + 1) = (100{,}000 - 1)$

$9(11{,}111) = 99{,}999$

c. If we substituted $x = 10$ into the product $(x-2)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ and computed the product, what number would result?

8(11,111,111) = 88,888,888

d. Multiply $(x-2)$ and $(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ and express your answer in standard form.

$x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x - 2x^7 - 2x^6 - 2x^5 - 2x^4 - 2x^3 - 2x^2 - 2x - 2$

$x^8 - x^7 - x^6 - x^5 - x^4 - x^3 - x^2 - x - 2$

Substitute $x = 10$ into your answer and see if you obtain the same result as you obtained in part (c).

$10^8 - 10^7 - 10^6 - 10^5 - 10^4 - 10^3 - 10^2 - 10 - 2 = 88,888,888$. *Yes, I get the same answer.*

e. Francois says $(x-9)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ must equal $x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ because when $x = 10$, multiplying by "$x - 9$" is the same as multiplying by 1.

i. Multiply $(x-9)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$

$x^8 - 8x^7 - 8x^6 - 8x^5 - 8x^4 - 8x^3 - 8x^2 - 8x - 9$

ii. Put $x = 10$ into your answer.

100,000,000 − 80,000,000 − 8,000,000 − 800,000 − 80,000 − 8,000 − 800 − 80 − 9

100,000,000 − 88,888,889 = 11,111,111

Is it the same as $x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ with $x = 10$?

Yes.

iii. Was Francois right?

No, just because it is true when x is 10, doesn't make it true for all real x. The two expressions are not algebraically equivalent.

Name ______________________________ Date ________________

1. Solve the following equations for x. Write your answer in set notation.

   a. $3x - 5 = 16$

   b. $3(x + 3) - 5 = 16$

   c. $3(2x - 3) - 5 = 16$

   d. $6(x + 3) - 10 = 32$

   e. Which two equations above have the same solution set? Write a sentence explaining how the properties of equality can be used to determine the pair without having to find the solution set for each.

2. Let c and d be real numbers.

   a. If $c = 42 + d$ is true, then which is greater: c or d, or are you not able to tell? Explain how you know your choice is correct.

   b. If $c = 42 - d$ is true, then which is greater: c or d, or are you not able to tell? Explain how you know your choice is correct.

3. If $a < 0$ and $c > b$, circle the expression that is greater:

$$a(b - c) \qquad \text{or} \qquad a(c - b)$$

   Use the properties of inequalities to explain your choice.

4. Solve for x in each of the equations or inequalities below, and name the property and/or properties used:

a. $\frac{3}{4}x = 9$

b. $10 + 3x = 5x$

c. $a + x = b$

d. $cx = d$

e. $\frac{1}{2}x - g < m$

f. $q + 5x = 7x - r$

g. $\frac{3}{4}(x + 2) = 6(x + 12)$

h. $3(5 - 5x) > 5x$

5. The equation $3x + 4 = 5x - 4$ has the solution set {4}.

   a. Explain why the equation $(3x + 4) + 4 = (5x - 4) + 4$ also has the solution set {4}.

b. In Part (a), the expression $(3x + 4) + 4$ is equivalent to the expression $3x + 8$. What is the definition of equivalent expressions? Why does changing an expression on one side of an equation to an equivalent expression leave the solution set unchanged?

c. When we square both sides of the original equation, we get the following new equation:

$$(3x + 4)^2 = (5x - 4)^2.$$

Show that 4 is still a solution to the new equation. Show that 0 is also a solution to the new equation but is not a solution to the original equation. Write a sentence that describes how the solution set to an equation may change when both sides of the equation are squared.

d. When we replace x by x^2 in the original equation, we get the following new equation:

$$3x^2 + 4 = 5x^2 - 4.$$

Use the fact that the solution set to the original equation is {4} to find the solution set to this new equation.

6. The Zonda Information and Telephone Company (ZI&T) calculates a customer's total monthly cell phone charge using the formula,

$$C = (b + rm)(1 + t),$$

where C is the total cell phone charge, b is a basic monthly fee, r is the rate per minute, m is the number of minutes used that month, and t is the tax rate.

Solve for m, the number of minutes the customer used that month.

7. Students and adults purchased tickets for a recent basketball playoff game. All tickets were sold at the ticket booth—season passes, discounts, etc. were not allowed.

   Student tickets cost $5 each, and adult tickets cost $10 each.
   A total of $4500 was collected. 700 tickets were sold.

   a. Write a system of equations that can be used to find the number of student tickets, s, and the number of adult tickets, a, that were sold at the playoff game.

   b. Assuming that the number of students and adults attending would not change, how much more money could have been collected at the playoff game if the ticket booth charged students and adults the same price of $10 per ticket?

   c. Assuming that the number of students and adults attending would not change, how much more money could have been collected at the playoff game if the student price was kept at $5 per ticket and adults were charged $15 per ticket instead of $10?

8. Alexus is modeling the growth of bacteria for an experiment in science. She assumes that there are B bacteria in a Petri dish at 12:00 noon. In reality, each bacterium in the Petri dish subdivides into two new bacteria *approximately* every 20 minutes. However, for the purposes of the model, Alexus assumes that each bacterium subdivides into two new bacteria *exactly* every 20 minutes.

   a. Create a table that shows the total number of bacteria in the Petri dish at 1/3 hour intervals for 2 hours starting with time 0 to represent 12:00 noon.

   b. Write an equation that describes the relationship between total number of bacteria T and time h in hours, assuming there are B bacteria in the Petri dish at $h = 0$.

   c. If Alexus starts with 100 bacteria in the Petri dish, draw a graph that displays the total number of bacteria with respect to time from 12:00 noon ($h = 0$) to 4:00 p.m. ($h = 4$). Label points on your graph at time $h = 0, 1, 2, 3, 4$.

d. For her experiment, Alexus plans to add an anti-bacterial chemical to the Petri dish at 4:00 p.m. that is supposed to kill 99.9% of the bacteria instantaneously. If she started with 100 bacteria at 12:00 noon, how many live bacteria might Alexus expect to find in the Petri dish right after she adds the anti-bacterial chemical?

9. Jack is 27 years older than Susan. In 5 years time he will be 4 times as old as her.

a. Find the present ages of Jack and Susan.

b. What calculations would you do to check if your answer is correct?

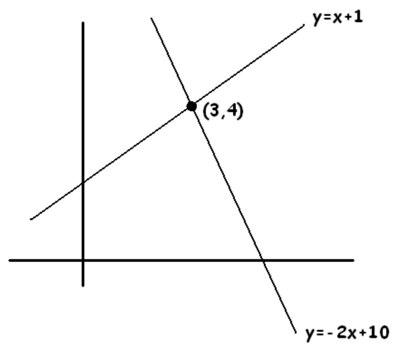
10.
a. Find the product: $(x^2 - x + 1)(2x^2 + 3x + 2)$.

b. Use the results of part (a) to factor $21{,}112$ as a product of a two-digit number and a three-digit number.

11. Consider the following system of equations with the solution *x* = 3, *y* = 4.

Equation A1: $y = x + 1$

Equation A2: $y = -2x + 10$



a. Write a unique system of two linear equations with the same solution set. This time make both linear equations have positive slope.

Equation B1: ________________

Equation B2: ________________

b. The following system of equations was obtained from the original system by adding a multiple of equation A2 to equation A1.

Equation C1: $y = x + 1$

Equation C2: $3y = -3x + 21$

What multiple of A2 was added to A1?

c. What is the solution to the system given in part (b)?

d. For any real number m, the line $y = m(x - 3) + 4$ passes through the point $(3,4)$.

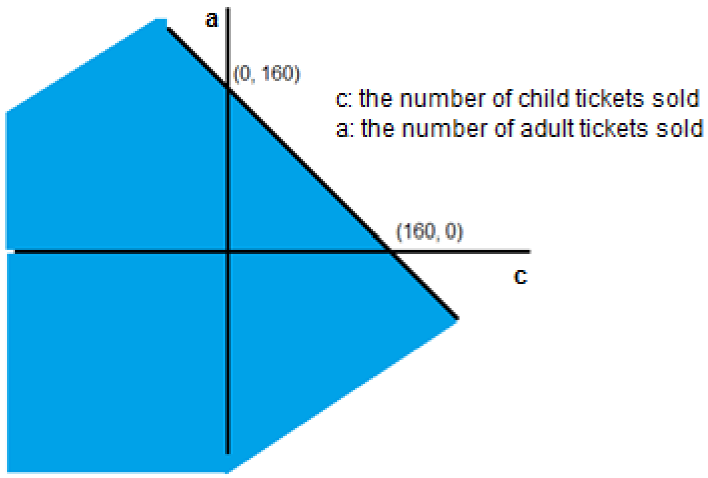
Is it certain, then, that the system of equations

Equation D1: $y = x + 1$

Equation D2: $y = m(x - 3) + 4$

has only the solution $x = 3$, $y = 4$? Explain.

12. The local theater in Jamie's home town has a maximum capacity of 160 people. Jamie shared with Venus the following graph and said that the shaded region represented all the possible combinations of adult and child tickets that could be sold for one show.



a. Venus objected and said there was more than one reason that Jamie's thinking was flawed. What reasons could Venus be thinking of?

b. Use equations, inequalities, graphs, and/or words to describe for Jamie the set of all possible combinations of adult and child tickets that could be sold for one show.

c. The theater charges \$9 for each adult ticket and \$6 for each child ticket. The theater sold 144 tickets for the first showing of the new release. The total money collected from ticket sales for that show was \$1164. Write a system of equations that could be used to find the number of child tickets and the number of adult tickets sold, and solve the system algebraically. Summarize your findings using the context of the problem.

A Progression Toward Mastery					
Assessment Task Item		**STEP 1** Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	**STEP 2** Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	**STEP 3** A correct answer with some evidence of reasoning or application of mathematics to solve the problem, OR an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	**STEP 4** A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	**a – d** A-REI.1	Student gave a short incorrect answer OR left the question blank.	Student showed at least one correct step, but the solution was incorrect.	Student solved the equation correctly (every step that was shown was correct) but did not express the answer as a solution set.	Student solved the equation correctly (every step that was shown was correct) AND expressed the answer as a solution set.
	e A-SSE.1b A-REI.3	Student did not answer OR answered incorrectly with something other than b and d.	Student answered b and d but did not demonstrate solid reasoning in the explanation.	Student answered b and d but made minor misstatements in the explanation.	Student answered b and d AND articulated solid reasoning in the explanation.
2	**a – b** A-CED.3	Student responded incorrectly OR left the question blank.	Student responded correctly that c must be greater but did not use solid reasoning to explain the answer.	Student responded correctly that c must be greater but gave an incomplete or slightly incorrect explanation of why.	Student responded correctly that c must be greater AND supported the statement with solid, well-expressed reasoning.
3	**a** A-SSE.1b	Student responded incorrectly OR left the question blank.	Student responded correctly by circling the expression on the left but did not use solid reasoning to explain the answer.	Student responded correctly by circling the expression on the left but gave limited explanation OR did not use the properties of inequality in the explanation.	Student responded correctly by circling the expression on the left AND gave a complete explanation that used the properties of inequality.

	b **A-SSE.1b**	Student was unable to respond to question <u>OR</u> left items blank.	Part (b) was answered incorrectly.	Student provided limited expression of the reasoning.	Student provided solid reasoning with examples.
4	**a – h** **A-REI.1** **A-REI.3**	Student answered incorrectly with no correct steps shown.	Student answered incorrectly but had one or more correct steps.	Student answered correctly but did not express the answer as a solution set.	Student answered correctly <u>AND</u> expressed the answer as a solution set.
5	**a** **A-REI.1**	Student did not answer <u>OR</u> demonstrated incorrect reasoning throughout.	Student demonstrated only limited reasoning.	Student demonstrated solid reasoning but fell short of a complete answer or made a minor misstatement in the answer.	Student answer was complete <u>AND</u> demonstrated solid reasoning throughout.
	b **A-REI.1**	Student did not answer <u>OR</u> did not demonstrate understanding of what the question was asking.	Student made more than one misstatement in the definition.	Student provided a mostly correct definition with a minor misstatement.	Student answered completely <u>AND</u> used a correct definition without error or mis-statement.
	c **A-REI.1**	Student made mistakes in both verifications and demonstrated incorrect reasoning <u>OR</u> left the question blank.	Student conducted both verifications but fell short of articulating reasoning to answer the question.	Student conducted both verifications <u>AND</u> articulated valid reasoning to answer the question but made a minor error in the verification or a minor misstatement in the explanation.	Student conducted both verifications without error <u>AND</u> articulated valid reasoning to answer the question.
	d **A-REI.1**	Student answered incorrectly <u>OR</u> does not answer.	Student identified one or both solutions but was unable to convey how the solutions could be found using the fact that 4 is a solution to the original equation.	Student identified only one solution correctly but articulated the reasoning of using the solution to the original equation to find the solution to the new equation.	Student identified both solutions correctly <u>AND</u> articulated the reasoning of using the solution to the original equation to find the solution to the new equation.
6	**A-CED.4**	Student did not answer <u>OR</u> showed no evidence of reasoning.	Student made more than one error in the solution process but showed some evidence of reasoning.	Student answer showed valid steps but with one minor error.	Student answered correctly.

7	**a – c** **A-CED.3**	Student was unable to answer any portion correctly.	Student answered one part correctly OR showed some evidence of reasoning in more than one part.	Student showed solid evidence of reasoning in every part but may have made minor errors.	Student answered every part correctly AND demonstrated and expressed valid reasoning throughout.
8	**a** **A-CED.2**	Student provided no table OR a table with multiple incorrect entries.	Data table is incomplete OR has more than one minor error.	Data table is complete but may have one error OR slightly inaccurate headings.	Data table is complete AND correct with correct headings.
	b **A-CED.2**	Student provided no equation OR an equation that did not represent exponential growth.	Student provided an incorrect equation but one that modeled exponential growth.	Student provided a correct answer in the form of $T = B(2)^{3h}$.	Student provided a correct answer in the form of $T = B8^{h}$ OR in more than one form such as $T = B(2)^{3h}$ and $T = B8^{h}$.
	c **A-CED.2**	Student provided no graph OR a grossly inaccurate graph.	Student provided a graph with an inaccurate shape but provided some evidence of reasoning in labeling the axes and/or data points.	Student created a graph with correct general shape but may have left off OR made an error on one or two axes or data points.	Student created complete graph with correctly labeled axes AND correctly labeled data points (or a data table) showing the values for h = 0, 1, 2, 3, 4.
	d **A-CED.2**	Student provided no answer OR an incorrect answer with no evidence of reasoning in arriving at the answer.	Student provided limited evidence of reasoning AND an incorrect answer.	Student answered that 409.6 bacteria would be alive.	Student answered that 410 or about 410 bacteria would be alive.
9	**a** **A-CED.1**	Student wrote incorrect equations OR did not provide equations.	Student answers were incorrect, but at least one of the equations was correct. Student may have made a gross error in the solution, made more than one minor error in the solution process, OR may have had one of the two equations incorrect.	Both equations were correct, but student made a minor mistake in finding the solution.	Both equations were correct AND student solved them correctly to arrive at the answer that Jack is 31 and Susan is 4.

	b **A-REI.3**	Student did not answer OR gave a completely incorrect answer.	Student articulated only one of the calculations correctly.	Student articulated the two calculations but with a minor mis-statement in one of the descriptions.	Student articulated both calculations correctly.
10	**a-b** **A-APR.1**	Student work is blank or demonstrates no understanding of multiplication of polynomials, nor how to apply part (a) to arrive at an answer for part (b).	Student made more than one error in the multiplication but demonstrated some understanding of multiplication of polynomials. Student may not have been able to garner or apply information from part *a* to use in answering part *b* correctly.	Student demonstrated the ability to multiply the polynomials (expressing the product as a sum of monomials with like terms combined) and to apply the structure from part *a* to solve part *b*. There may be minor errors.	Student demonstrated the ability to multiply the polynomials (expressing the product as a sum of monomials with like terms combined) and to apply the structure from part *a* to solve part *b* as: $91(232)$.
11	**a** **A-REI.6**	Student was unable to demonstrate the understanding that two equations with (3, 4) as a solution are needed.	Student provided two equations that have (3, 4) as a solution (or attempted to provide such equations) but made one or more errors. Student may have provided an equation with a negative slope.	Student showed one minor error in the answer but attempted to provide two equations both containing (3, 4) as a solution and both with positive slope.	Student provided two equations both containing (3, 4) as a solution and both with positive slope.
	b **A-REI.6**	Student was unable to identify the multiple correctly.	Student identified the multiple as 3.	N/A	Student correctly identified the multiple as 2.
	c **A-REI.6**	Student was unable to demonstrate even a partial understanding of how to find the solution to the system.	Student showed some reasoning required to find the solution but made multiple errors.	Student made a minor error in finding the solution point.	Student successfully identified the solution point as $(3, 4)$.
	d **A-REI.5** **A-REI.6** **A-REI.10**	Student was unable to answer or to support the answer with any solid reasoning.	Student concluded yes or no but was only able to express limited reasoning in support of the answer.	Student correctly explained that all the systems would have the solution point $(3, 4)$ but incorrectly assumed this is true	Student correctly explained that while in most cases this is true, that if $m = 1$, the two lines are coinciding lines,

				for all cases of m.	resulting in a solution set consisting of all the points on the line.
12	**a** **MP.2** **A-REI.12**	Student was unable to articulate any sound reasons.	Student was only able to articulate one sound reason.	Student provided two sound reasons but made minor errors in the expression of reasoning.	Student was able to articulate at least 2 valid reasons. Valid reasons include the following: the graph assumes x could be less than zero, the graph assumes y could be less than zero, the graph assumes a and b could be non-whole numbers, the graph assumes 160 children could attend with no adults.
	b **A-CED.2** **A-REI.10** **A-REI.12**	Student was unable to communicate a relevant requirement of the solution set.	Student provided a verbal description that lacked precision and accuracy but demonstrated some reasoning about the solution within the context of the problem.	Student made minor errors in communicating the idea that both (a) and (b) must be whole numbers whose sum is less than or equal to 160.	Student communicated effectively that both (a) and (b) must be whole numbers whose sum is less than or equal to 160.
	c **A-CED.2** **A-REI.6**	Student was unable to demonstrate any substantive understanding in how to create the equations and solve the system of equations.	Student made multiple errors in the equations and/or solving process but demonstrated some understanding of how to create equations to represent a context and/or solve the system of equations.	Student made minor errors in the equations but solved the system accurately, or the student created the correct equations but made a minor error in solving the system of equations.	Student correctly wrote the equations to represent the system. Student solved the system accurately and summarized by defining or describing the values of the variable in the context of the problem: that there were 100 adult tickets and 44 child tickets sold.

Name ______________________________ Date ____________________

1. Solve the following equations for x. Write your answer in set notation.

   a. $3x - 5 = 16$

   $3x = 21$ Solution set: $\{7\}$

   $x = 7$

   b. $3(x + 3) - 5 = 16$

   $3x + 9 - 5 = 16$ Solution set: $\{4\}$

   $3x = 12$

   $x = 4$

   c. $3(2x - 3) - 5 = 16$

   $6x - 9 - 5 = 16$ Solution set: $\{5\}$

   $6x - 14 = 16$

   $6x = 30$

   $x = 5$

   d. $6(x + 3) - 10 = 32$

   $6x + 18 - 10 = 32$ Solution set: $\{4\}$

   $6x = 24$

   $x = 4$

   e. Which two equations above have the same solution set? Write a sentence explaining how the properties of equality can be used to determine the pair without having to find the solution set for each.

   Problems (b) and (d) have the same solution set. The expressions on each side of the equal sign for d are twice those for b. So, if (left side) = (right side) is true for only some x-values, then 2(left side) = 2(right side) will be true for exactly the same x-values. Or simply, applying the multiplicative property of equality does not change the solution set.

2. Let c and d be real numbers.

a. If $c = 42 + d$ is true, then which is greater: c or d or are you not able to tell? Explain how you know your choice is correct.

c must be greater because c is always 42 more than d.

b. If $c = 42 - d$ is true, then which is greater: c or d or are you not able to tell? Explain how you know your choice is correct.

There is no way to tell. We only know that the sum of c and d is 42. If d were 10, c would be 32 and, therefore, greater than d. But if d were 40, c would be 2 and, therefore, less than d.

3. If $a < 0$ and $c > b$, circle the expression that is greater:

$$a(b-c) \quad \text{or} \quad a(c-b)$$

Use the properties of inequalities to explain your choice.

Since c > b, *it follows that 0 > b − c.* *So (b−c) is negative.* *And since a < 0, a is negative.* *And the product of two negatives will be positive.*	*Since c > b,* *it follows that c − b > 0.* *So (c−b) is positive.* *And since a is negative,* *the product of a(c−b) is negative.* *So, a(c−b) < a(b−c).*

4. Solve for x in each of the equations or inequalities below and name the property and/or properties used:

a. $\frac{3}{4}x = 9$

$x = 9\ (3/4)$ Multiplication Property of Equality

$x = 12$

b. $10 + 3x = 5x$

$10 = 2x$ Addition Property of Equality

$5 = x$

c. $a + x = b$

$x = b - a$ Addition Property of Equality

d. $cx = d$

$x = d/c,\ c \neq 0$ Multiplication Property of Equality

e. $\frac{1}{2}x - g < m$

$\frac{1}{2}x < m + g$ Addition Property of Equality

$x < 2(m+g)$ Multiplication Property of Equality

f. $q + 5x = 7x - r$

$q + r = 2x$ Addition Property of Equality

$(q + r)/2 = x$ Multiplication property of Equality

g. $\frac{3}{4}(x+2) = 6(x+12)$

3(x+2) = 24(x + 12)	*Multiplication Property of Equality*
3x + 6 = 24x + 288	*Distributive Property*
-282/21 = x	*Addition Property of Equality*
-94/7 = x	

h. $3(5 - 5x) > 5x$

15 − 15x > 5x	*Distributive Property*
15 > 20 x	*Addition Property of Inequality*
¾ > x	

5. The equation, $3x + 4 = 5x - 4$, has the solution set {4}.

a. Explain why the equation, $(3x + 4) + 4 = (5x - 4) + 4$, also has the solution set {4}.

Since the new equation can be created by applying the additive property of equality, the solution set does not change.

Or:

Each side of this equation is 4 more than the sides of the original equation. Whatever value(s) make 3x + 4 = 5x − 4 true would also make 4 more than 3x + 4 equal to 4 more than 5x − 4.

b. In Part (a), the expression $(3x + 4) + 4$ is equivalent to the expression $3x + 8$. What is the definition of equivalent algebraic expressions? Describe why changing an expression on one side of an equation to an equivalent expression leaves the solution set unchanged?

Algebraic expressions are equivalent if (possibly repeated) use of the Distributive, Associative, and Commutative Properties, and/or the properties of rational exponents can be applied to one expression to convert it to the other expression.

When two expressions are equivalent, assigning the same value to x in both expressions will give an equivalent numerical expression which then evaluates to the same number. Therefore, changing the expression to something equivalent will not change the truth value of the equation once values are assigned to x.

c. When we square both sides of the original equation, we get the following new equation:

$$(3x + 4)^2 = (5x - 4)^2.$$

Show that 4 is still a solution to the new equation. Show that 0 is also a solution to the new equation but is not a solution to the original equation. Write a sentence that describes how the solution set to an equation may change when both sides of the equation are squared.

$(3 \cdot 4 + 4)^2 = (5 \cdot 4 - 4)^2$ gives $16^2 = 16^2$ which is true.

$(3 \cdot 0 + 4)^2 = (5 \cdot 0 - 4)^2$ gives $4^2 = (-4)^2$ which is true.

But, $(3 \cdot 0 + 4) = (5 \cdot 0 - 4)$ gives $4 = -4$ which is false.

When both sides are squared, you might introduce new numbers to the solution set because statements like $4 = -4$ are false, but statements like $4^2 = (-4)^2$ are true.

d. When we replace x by x^2 in the original equation, we get the following new equation:

$$3x^2 + 4 = 5x^2 - 4.$$

Use the fact that the solution set to the original equation is {4} to find the solution set to this new equation.

Since the original equation $3x + 4 = 5x - 4$ was true when $x = 4$, the new equation $3x^2 + 4 = 5x^2 - 4$ should be true when $x^2 = 4$. And, $x^2 = 4$ when $x = 2$ or when $x = -2$, so the solution set to the new equation is $\{-2, 2\}$.

6. The Zonda Information and Telephone Company (ZI&T) calculates a customer's total monthly cell phone charge using the formula,

$$C = (b + rm)(1 + t),$$

where C is the total cell phone charge, b is a basic monthly fee, r is the rate per minute, m is the number of minutes used that month, and t is the tax rate.

Solve for m, the number of minutes the customer used that month.

$$C = b + bt + rm + rmt$$

$$C - b - bt = m(r + rt)$$

$$\frac{C - b - bt}{r + rt} = m \qquad t \neq -1, \quad r \neq 0$$

7. Students and adults purchased tickets for a recent basketball playoff game. All tickets were sold at the ticket booth—season passes, discounts, etc. were not allowed.

Student tickets cost $5 each, and adult tickets cost $10 each.
A total of $4500 was collected. 700 tickets were sold.

a. Write a system of equations that can be used to find the number of student tickets, *s*, and the number of adult tickets, *a*, that were sold at the playoff game.

$$\begin{cases} 5s + 10a = 4500 \\ s + a = 700 \end{cases}$$

b. Assuming that the number of students and adults attending would not change, how much more money could have been collected at the playoff game if the ticket booth charged students and adults the same price of $10 per ticket?

$700 \times \$10 = \$7000$

$\$7000 - \$4500 = \$2500$ more.

c. Assuming that the number of students and adults attending would not change, how much more money could have been collected at the playoff game if the student price was kept at $5 per ticket and adults were charged $15 per ticket instead of $10?

First solve for a & s

$$\begin{aligned} 5s + 10a &= 4500 \\ -5s - 5a &= -3500 \end{aligned}$$

$5a = 1000$
$a = 200$
$s = 500$

$\$5(500) + \$15(200) = \$5500$

$1000 more

or

$5 more per adult tkt
$200 \times \$5$
$= \$1000$ more

8. Alexus is modeling the growth of bacteria for an experiment in science. She assumes that there are B bacteria in a Petri dish at 12:00 noon. In reality, each bacterium in the Petri dish subdivides into two new bacteria *approximately* every 20 minutes. However, for the purposes of the model, Alexus assumes that each bacterium subdivides into two new bacteria *exactly* every 20 minutes.

   a. Create a table that shows the total number of bacteria in the Petri dish at 1/3 hour intervals for 2 hours starting with time 0 to represent 12:00 noon.

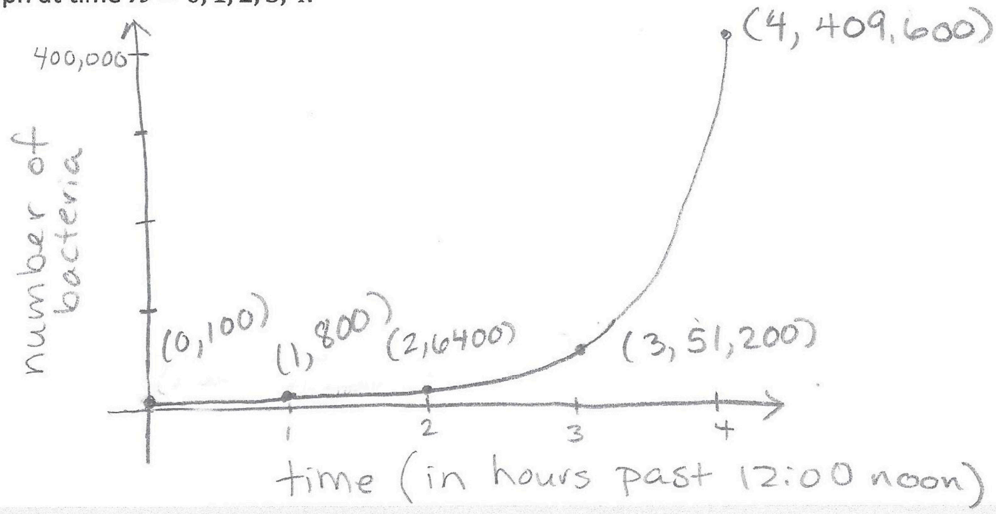
time	number of bacteria
0	B
1/3 hr	2 B
2/3 hr	4 B
1 hr	8 B
1 1/3 hr	16 B
1 2/3 hr	32 B
2 hr	64 B

   b. Write an equation that describes the relationship between total number of bacteria T and time h in hours, assuming there are B bacteria in the Petri dish at $h = 0$.

$$T = B(2)^{3h}$$

or $T = B \cdot 8^h$

   c. If Alexus starts with 100 bacteria in the Petri dish, draw a graph that displays the total number of bacteria with respect to time from 12:00 noon ($h = 0$) to 4:00 p.m. ($h = 4$). Label points on your graph at time $h = 0, 1, 2, 3, 4$.



d. For her experiment, Alexus plans to add an anti-bacterial chemical to the Petri dish at 4:00 p.m. that is supposed to kill 99.9% of the bacteria instantaneously. If she started with 100 bacteria at 12:00 noon, how many live bacteria might Alexus expect to find in the Petri dish right after she adds the anti-bacterial chemical?

$(1-.999)\,409{,}600 = 409.6$

about 410 live bacteria.

9. Jack is 27 years older than Susan. In 5 years time he will be 4 times as old as her.

a. Find Jack and Susan's present age.

$$\begin{cases} J = S+27 \\ J+5 = 4(S+5) \end{cases}$$

$S+27+5 = 4S+20$

$S+32 = 4S+20$

$12 = 3S \qquad S=4$

$J = 4+27$

$J = 31$

Jack is 31 and Susan is 4.

b. What calculations would you do to check if your answer is correct?

Is Jack's age − Susan's age = 27?
add 5 years to Jack & Susan's ages and see if that makes Jack 4 times as old as Susan.

10.

a. Find the product: $(x^2 - x + 1)(2x^2 + 3x + 2)$

$2x^4 + 3x^3 + 2x^2 - 2x^3 - 3x^2 - 2x + 2x^2 + 3x + 2$

$2x^4 + x^3 + x^2 + x + 2$

b. Use the results of part (a) to factor 21,112 as a product of a two-digit number and a three-digit number.

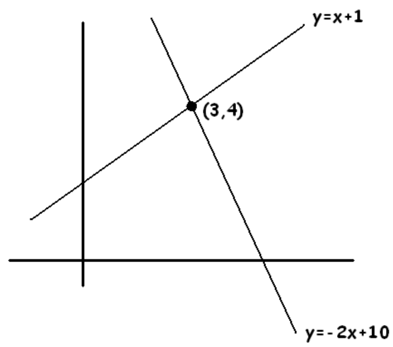
$(100 - 10 + 1)(200 + 30 + 2)$

$(91)(232)$

11. Consider the following system of equations with the solution $x = 3$, $y = 4$.

Equation A1: $y = x + 1$

Equation A2: $y = -2x + 10$



a. Write a unique system of two linear equations with the same solution set. This time make both linear equations have positive slope.

Equation B1: $y = 4/3\ x$

Equation B2: $y = x + 1$

b. The following system of equations was obtained from the original system by adding a multiple of equation A2 to equation A1.

Equation C1: $y = x + 1$

Equation C2: $3y = -3x + 21$

What multiple of A2 was added to A1?

2

c. What is the solution to the system given in part (b)?

(3, 4)

d. For any real number m, the line $y = m(x - 3) + 4$ passes through the point $(3,4)$.

Is it certain, then, that the system of equations:

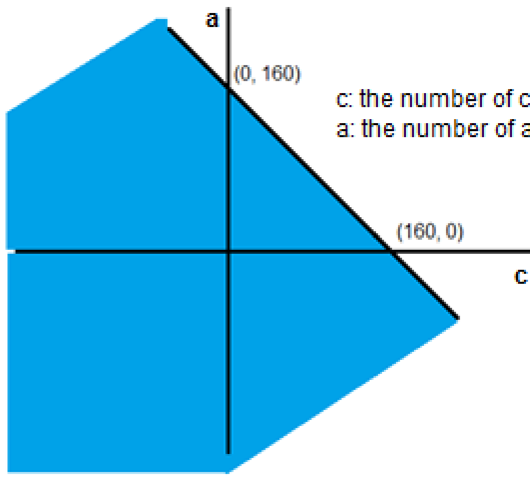
Equation D1: $y = x + 1$

Equation D2: $y = m(x - 3) + 4$

has only the solution $x = 3$, $y = 4$? Explain.

No. If m = 1, then the two lines have the same slope. Since both lines pass through the point (3, 4), and the lines are parallel, they, therefore, coincide. There are infinite solutions. The solution set is all the points on the line. Any other non-zero value of m would create a system with the only solution of (3, 4).

12. The local theater in Jamie's home town has a maximum capacity of 160 people. Jamie shared with Venus the following graph and said that the shaded region represented all the possible combinations of adult and child tickets that could be sold for one show.

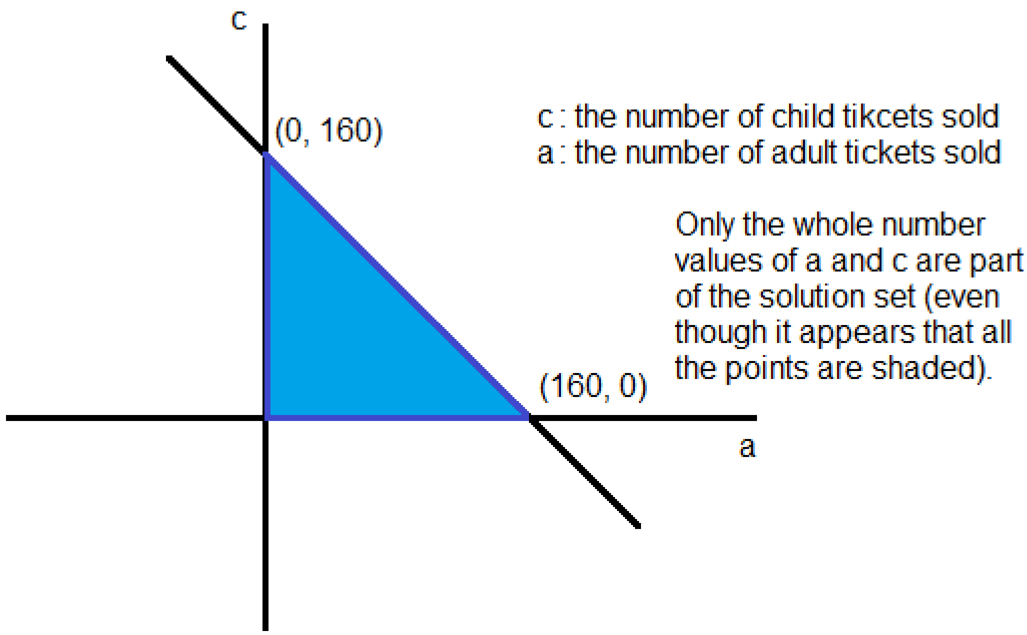


a. Venus objected and said there was more than one reason that Jamie's thinking was flawed. What reasons could Venus be thinking of?

1. *The graph implies that the number of tickets sold could be a fractional amount, but really it only makes sense to sell whole number tickets. X and y must be whole numbers.*
2. *The graph also shows that negative ticket amounts could be sold which does not make sense.*

b. Use equations, graphs and/or words to describe for Jamie the set of all possible combinations of adult and child tickets that could be sold for one show.

The system would be $\begin{cases} a + c \leq 160 \\ a \geq 0 \\ c \geq 0 \end{cases}$ where a and c are whole numbers.



c. The theater charges \$9 for each adult ticket and \$6 for each child ticket. The theater sold 144 tickets for the first showing of the new release. The total money collected from ticket sales for that show was \$1164. Write a system of equations that could be used to find the number of child tickets and the number of adult tickets sold, and solve the system algebraically. Summarize your findings using the context of the problem.

a: the number of adult tickets sold (must be a whole number)

c: the number of child tickets sold (must be a whole number)

$$\begin{cases} 9a + 6c = 1164 \\ a + c = 144 \end{cases}$$

$9a + 6c = 1164$

$-6a - 6c = -864$

$3a = 300$ $a = 100$, $c = 44$.

In all, 100 adult tickets and 44 child tickets were sold.