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Ratios and Unit Rates

[1] Each lesson is ONE day and ONE day is considered a 45 minute period.

Grade 6 • Module 1

Ratios and Unit Rates

OVERVIEW

In this module, students are introduced to the concepts of ratio and rate. Their previous experience solving problems involving multiplicative comparisons, such as *"Max has three times as many toy cars as Jack,"* (**4.OA.2**) serves as the conceptual foundation for understanding ratios as a multiplicative comparison of two or more numbers used in quantities or measurements (**6.RP.1**). Students develop fluidity in using multiple forms of ratio language and ratio notation. They construct viable arguments and communicate reasoning about ratio equivalence as they solve ratio problems in real world contexts (**6.RP.3**). As the first topic comes to a close, students develop a precise definition of the value of a ratio $a:b$, where $b \neq 0$ as the value a/b, applying previous understanding of fraction as division (**5.NF.3**). They can then formalize their understanding of equivalent ratios as ratios having the same value.

With the concept of ratio equivalence formally defined, students explore collections of equivalent ratios in real world contexts in Topic B. They build ratio tables and study their additive and multiplicative structure (**6.RP.3a**). Students continue to apply reasoning to solve ratio problems while they explore representations of collections of equivalent ratios and relate those representations to the ratio table (**6.RP.3**). Building on their experience with number lines, students represent collections of equivalent ratios with a double number line model. They relate ratio tables to equations using the value of a ratio defined in Topic A. Finally, students expand their experience with the coordinate plane (**5.G.1**, **5.G.2**) as they represent collections of equivalent ratios by plotting the pairs of values on the coordinate plane. The Mid-Module Assessment follows Topic B.

In Topic C, students build further on their understanding of ratios and the value of a ratio as they come to understand that a ratio of 5 miles to 2 hours corresponds to a rate of 2.5 miles per hour, where the *unit rate* is the numerical part of the rate, 2.5, and *miles per hour* is the newly formed unit of measurement of the rate (**6.RP.2**). Students solve unit rate problems involving unit pricing, constant speed, and constant rates of work (**6.RP.3b**). They apply their understanding of rates to situations in the real world. Students determine unit prices and use measurement conversions to comparison shop, and decontextualize constant speed and work situations to determine outcomes. Students combine their new understanding of rate to connect and revisit concepts of converting among different-sized standard measurement units (**5.MD.1**). They then expand upon this background as they learn to manipulate and transform units when multiplying and dividing quantities (**6.RP.3d**). Topic C culminates as students interpret and model real-world scenarios through the use of unit rates and conversions.

In the final topic of the module, students are introduced to percent and find percent of a quantity as a *rate per 100*. Students understand that *N* percent of a quantity has the same value as *N/100* of that quantity. Students express a fraction as a percent, and find a percent of a quantity in real-world contexts. Students learn to express a ratio using the language of percent and to solve percent problems by selecting from familiar representations, such as tape diagrams and double number lines, or a combination of both (**6.RP.3c**). An End-of-Module Assessment follows Topic D.

Focus Standards

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.2 Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b ≠ 0*, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[2]

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

c. Find a percent of a quantity as a rate per *100* (e.g., *30%* of a quantity means *30/100* times the quantity); solve problems involving finding the whole, given a part and the percent.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Foundational Standards

Use the four operations with whole numbers to solve problems.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[3]

[2] Expectations for unit rates in this grade are limited to non-complex fractions.
[3] See Glossary, Table 2.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.3 Interpret a fraction as division of the numerator by the denominator (*a/b = a ÷ b*). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

Convert like measurement units within a given measurement system.

5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students make sense of and solve real world and mathematical ratio, rate, and percent problems using representations, such as tape diagrams, ratio tables, the coordinate plane, and/or double number line diagrams. They identify and explain the correspondences between the verbal descriptions and their representations and articulate how the representation depicts the relationship of the quantities in the problem. Problems include ratio problems involving the comparison of three quantities, multistep changing ratio problems, using a given ratio to find associated ratios, and constant rate problems including two or more people or machines working together.

MP.2 **Reason abstractly and quantitatively.** Students solve problems by analyzing and comparing ratios and unit rates given in tables, equations, and graphs. Students decontextualize a given constant speed situation, representing symbolically the quantities involved with the formula, *distance = rate x time*.

MP.5 **Use appropriate tools strategically.** Students become proficient using a variety of representations that are useful in reasoning with rate and ratio problems such as tape diagrams, double line diagrams, ratio tables, a coordinate plane and equations. They then use judgment in selecting appropriate tools as they solve ratio and rate problems.

MP.6 **Attend to precision.** Students define and distinguish between ratio, the value of a ratio, a unit rate, a rate unit, and a rate. Students use precise language and symbols to describe ratios and rates. Students learn and apply the precise definition of percent.

MP.7 **Look for and make use of structure.** Students recognize the structure of equivalent ratios in solving word problems using tape diagrams. Students identifying the structure of a ratio table and use it to find missing values in the table. Students make use of the structure of division and ratios to model *5 miles/2 hours* as a quantity *2.5 mph*.

Terminology

New or Recently Introduced Terms

- **Ratio** (A pair of nonnegative numbers, *A:B*, where both are not zero, and that are used to indicate that there is a relationship between two quantities such that when there are *A* units of one quantity, there are *B* units of the second quantity.)
- **Rate** (A rate indicates, for a proportional relationship between two quantities, how many units of one quantity there are for every 1 unit of the second quantity. For a ratio of *A:B* between two quantities, the rate is *A/B* units of the first quantity per unit of the second quantity.)
- **Unit Rate** (The numeric value of the rate, e.g., in the rate 2.5 mph, the unit rate is 2.5.)
- **Value of a Ratio** (For the ratio *A:B*, the value of the ratio is the quotient *A/B*.)
- **Equivalent Ratios** (Ratios that have the same value.)
- **Percent** (Percent of a quantity is a rate per 100.)
- **Associated Ratios** (e.g., if a popular shade of purple is made by mixing 2 cups of blue paint for every 3 cups of red paint, not only can we say that the ratio of blue paint to red paint in the mixture is 2:3, but we can discuss associated ratios such as the ratio of cups of red paint to cups of blue paint, the ratio of cups of blue paint to total cups of purple paint, the ratio of cups of red paint to total cups of purple paint, etc.)
- **Double Number Line** (See example under Suggested Tools and Representations.)
- **Ratio Table** (A table listing pairs of numbers that form equivalent ratios; see example under Suggested Tools and Representations.)

Familiar Terms and Symbols[4]

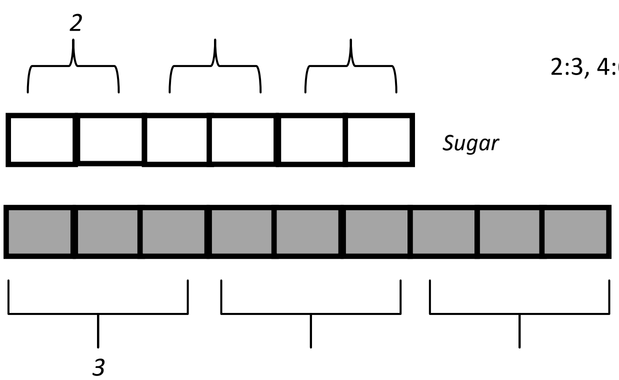
- Convert
- Tape Diagram
- Coordinate Plane
- Equation

Suggested Tools and Representations

- Tape Diagrams (See example below)
- Double Number Line Diagrams (See example below)
- Ratio Tables (See example below)
- Coordinate Plane (See example below)

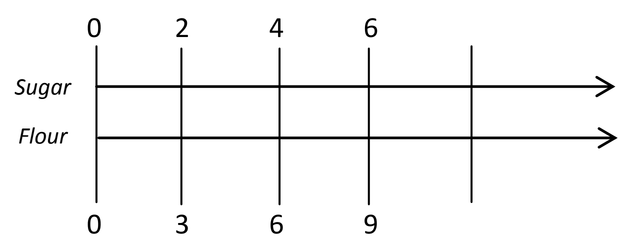
Representing Equivalent Ratios for a cake recipe
that uses 2 cups of sugar for every 3 cups of flour

Tape Diagram



2:3, 4:6, 6:9

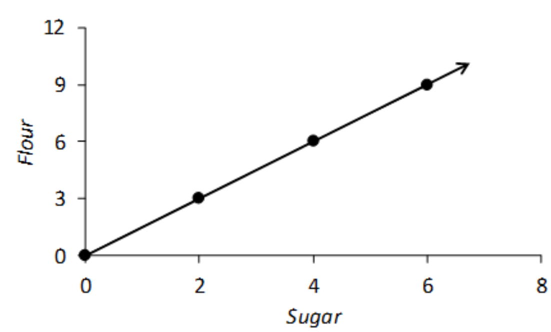
Double Number Line



Ratio Table

Sugar	*Flour*
2	3
4	6
6	9

Coordinate Plane



[4] These are terms and symbols students have seen previously.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	6.RP.1, 6.RP.3 (Stem Only), 6.RP.3a
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	6.RP.1, 6.RP.2, 6.RP.3

Topic A:

Representing and Reasoning About Ratios

6.RP.1, 6.RP.3a

Focus Standard:	6.RP.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
	6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
Instructional Days:	8	
Lessons 1–2:	Ratios (S/E)[1]	
Lessons 3–4:	Equivalent Ratios (P)	
Lessons 5–6:	Solving Problems by Finding Equivalent Ratios (P)	
Lesson 7:	Associated Ratios and the Value of a Ratio (P)	
Lesson 8:	Equivalent Ratios Defined Through the Value of a Ratio (M)	

[1] Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

In Topic A, students are introduced to the concepts of ratios. Their previous experience solving problems involving multiplicative comparisons, such as *"Max has three times as many toy cars as Jack"* (**4.OA.2**), serves as the conceptual foundation for understanding ratios as a multiplicative comparison of two or more numbers used in quantities or measurements (**6.RP.1**). In the first two lessons, students develop fluidity in using multiple forms of ratio language and ratio notation as they read about or watch video clips about ratio relationships and then discuss and model the described relationships. Students are prompted to think of, describe, and model ratio relationships from their own experience. Similarly, Lessons 3 and 4 explore the idea of equivalent ratios. Students read about or watch video clips about situations that call for establishing an equivalent ratio. Students discuss and model the situations to solve simple problems of finding one or more equivalent ratios.

The complexity of problems increases as students are challenged to find values of quantities in ratio given the total desired quantity or given the difference between the two quantities. *For example, "If the ratio of boys to girls in the school is 2:3, find the number of girls if there are 300 more girls than boys."* As the first topic comes to a close, students develop a precise definition of the *value of a ratio a:b*, where $b \neq 0$, as the value a/b, applying previous understanding of fraction as division (**5.NF.3**). Students are then challenged to express their understanding of ratio equivalence using the newly defined term, value of a ratio; they conclude that equivalent ratios are ratios having the same value.

Lesson 1: Ratios

Student Outcomes

- Students understand that a *ratio* is an ordered pair of non-negative numbers, which are not both zero. Students understand that a ratio is often used instead of describing the first number as a multiple of the second.
- Students use the precise language and notation of ratios (e.g., $3:2$, 3 to 2). Students understand that the order of the pair of numbers in a ratio matters and that the description of the ratio relationship determines the correct order of the numbers. Students conceive of real-world contextual situations to match a given ratio.

Lesson Notes

The first two lessons of this module will develop the students' understanding of the term *ratio*. A ratio is always a pair of numbers, such as $2:3$ and never a pair of quantities such as 2 cm : 3 sec. Keeping this straight for students will require teachers to use the term ratio correctly and consistently.". Students will be required to separately keep track of the units in a word problem. To help distinguish between ratios and statements about quantities that define ratios, we use the term *ratio relationship* to describe a phrase in a word problem that indicates a ratio. Typical examples of ratio relationship descriptions include "3 cups to 4 cups," "5 miles in 4 hours," etc. The ratios for these ratio relationships are $3:4$ and $5:4$, respectively.

Classwork

Example 1 (15 minutes)

Read the example aloud.

> **Example 1**
>
> **The coed soccer team has four times as many boys on it as it has girls. We say the ratio of the number of boys to the number of girls on the team is $4:1$. We read this as "four to one."**

- Let's create a table to show how many boys and how many girls on are on the team.

Create a table like the one shown below to show possibilities of the number of boys and girls on the soccer team. Have students copy the table into their student packet.

# of Boys	# of Girls	Total # of Players
4	1	5

- So, we would have four boys and one girl on the team for a total of five players. Is this big enough for a team?
  - *Adult teams require 11 players, but youth teams may have fewer. There is no right or wrong answer; just encourage the reflection on the question, thereby connecting their math work back to the context.*
- What are some other options that show four times as many boys as girls or a ratio of boys to girls of 4 to 1?
  - *Have students add each option given to their table.*

# of Boys	# of Girls	Total # of Players
4	1	5
8	2	10
12	3	15

- From the table, we can see that there are four boys for every one girl on the team.

Read the example aloud.

Suppose the ratio of number of boys to number of girls on the team is $3:2$.

Create a table like the one shown below to show possibilities of the number of boys and girls on the soccer team. Have students copy the table into their student packets.

# of Boys	# of Girls	Total # of Players
3	2	5

- What are some other options that show that there are three boys for every two girls on the team?

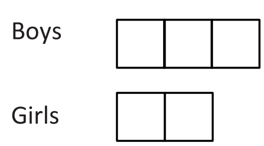
# of Boys	# of Girls	Total # of Players
3	2	5
6	4	10
9	6	15

- I can't say there are 3 times as many boys as girls. What would my multiplicative value have to be? There are ____ as many boys as girls.

Encourage the students to articulate their thoughts, guiding them to say there are $\frac{3}{2}$ as many boys as girls.

- Can you visualize $\frac{3}{2}$ as many boys as girls?

- Can we make a tape diagram (or bar model) that shows that there are $\frac{3}{2}$ as many boys as girls?



- Which description makes the relationship easier to visualize: saying the ratio is 3 to 2 or saying there are 3 halves as many boys as girls?
  - *There is no right or wrong answer. Have students explain why they picked their choices.*

Example 2 (8 minutes): Class Ratios

Discussion (4 minutes)

Direct students:

- Find the ratio of boys to girls in our class.
- Raise your hand when you know: What is the ratio of boys to girls in our class?
- How can we say this as a multiplicative comparison without using ratios? Raise your hand when you know. *Allow for choral response when all hands are raised.*
- Write the ratio of number of boys to number of girls in your student packet under Example 2, Question 1.
- Compare your answer with your neighbor's answer. Does everyone's ratio look exactly the same?
  - Allow for discussion of differences in what students wrote. Communicate the following in the discussions:
    1. It is ok to use either the colon symbol or the word 'to' between the two numbers of the ratio.
    2. The ratio itself does not have units or descriptive words attached.
- Raise your hand when you know: What is the ratio of number of girls to number of boys in our class?
- Write the ratio down in your packet as number 2.
- Is the ratio of number of girls to number of boys the same as the ratio of number of boys to number of girls?
  - *Unless in this case there happens to be an equal number of boys and girls, then no, the ratios are not the same. Indicate that order matters.*
- Is this an interesting multiplicative comparison for this class? Is it worth commenting on in our class? If our class had 15 boys and 5 girls, might it be a more interesting observation?

For the exercise below, choose a way for students to indicate that they identify with the first statement (e.g., standing up or raising a hand). After each pair of statements below, have students create a ratio of the first statement to the second statement verbally, in writing, or both. Consider following each pair of statements with a discussion of whether it seems like an interesting ratio to discuss. Or alternatively, when you have finished all of these examples, ask students which ratio they found most interesting.

Students record a ratio for each of the examples you provide:

1. You traveled out of state this summer.
2. You did not travel out of state this summer.
3. You have at least one sibling.
4. You are an only child.
5. Your favorite class is math.
6. Your favorite class is not math.

Example 2: Class Ratios

Record a ratio for each of the examples the teacher provides.

1. *Answers will vary. One example is* $12:10$.
2. *Answers will vary. One example is* $10:12$.
3. *Answers will vary. One example is* $7:15$.
4. *Answers will vary. One example is* $15:7$.
5. *Answers will vary. One example is* $11:11$.
6. *Answers will vary. One example is* $11:11$.

Exercise 1 (2 minutes)

Have students look around the classroom to think of their own ratios. Have students create written ratio statements that represent their ratios in one of the summary forms.

Exercise 1

My own ratio compares *number of students wearing jeans* **to** *number of students not wearing jeans*.

My ratio $16:6$

Exercise 2 (10 minutes)

MP.6

Students work with partners to write ratios in words that could be represented by each ratio given. Encourage students to be precise about the order in which the quantities are stated (emphasizing that order matters) and about the quantities being compared. That is, instead of saying the ratio of boys to girls, encourage them to say, the ratio of the number of boys to the number of girls. After students develop the capacity to be very precise about the quantities in the ratio, it is appropriate for them to abbreviate their communication in later lessons. Just be sure their abbreviations still accurately convey the meaning of the ratio in the correct order.

Exercise 2

Using words, describe a ratio that represents each ratio below.

a. 1 to 12 *for every one year, there are twelve months*

b. $12:1$ *for every twelve months, there is one year*

c. 2 to 5 *for every* 2 *days of non-school days in a week, there are five school days*

d. 5 to 2 *for every* 5 *female teachers I have, there are* 2 *male teachers*

e. $10:2$ *for every* 10 *toes, there are* 2 *feet*

f. $2:10$ *for every* 2 *problems I can finish, there are* 10 *minutes that pass*

After completion, invite sharing and explanations to the chosen answers.

Point out the difference between ratios like, "for every one year, there are twelve months" and ratios like, "for every 5 female teachers I have, there are 2 male teachers." The first type represents a constant relationship that will remain true as the number of years or months increases and the second one is somewhat arbitrary and will not remain true if the number of teachers increases.

Closing (5 minutes)

Provide students with this description:

A ratio is an ordered pair of non-negative numbers, which are not both zero. The ratio is denoted $A: B$ or A to B to indicate the order of the numbers. The number A is first, and the number B is second.

- What is a ratio? Can you verbally describe a ratio in your own words using this description?
- How do we write ratios?
  - *A colon B (A: B) or A 'to' B.*
- What are two quantities you would love to have in a ratio of $5: 2$ but hate to have in a ratio of $2: 5$?

Lesson Summary

A ratio is an ordered pair of non-negative numbers, which are not both zero.

The ratio is written $A: B$ or A *to* B to indicate the order of the numbers. The number A is first, and the number B is second.

The order of the numbers is important to the meaning of the ratio. Switching the numbers changes the relationship. The description of the ratio relationship tells us the correct order for the numbers in the ratio.

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 1: Ratios

Exit Ticket

1. Write a ratio for the following description: Kaleel made three times as many baskets as John during basketball practice.

2. Describe a situation that could be modeled with the ratio $4:1$.

3. Write a ratio for the following description: For every 6 cups of flour in a bread recipe, there are 2 cups of milk.

Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

1. **Write a ratio for the following description: Kaleel made three times as many baskets as John during basketball practice.**

   A ratio of $3:1$ or 3 to 1 can be used.

2. **Describe a situation that could be modeled with the ratio $4:1$.**

   Answers will vary but could include the following: For every four teaspoons of cream in a cup of tea, there is one teaspoon of honey.

3. **Write a ratio for the following description: For every 6 cups of flour in a bread recipe, there are 2 cups of milk.**

   A ratio of $6:2$ or 6 to 2 can be used, or students might recognize and suggest the equivalent ratio of $3:1$.

Problem Set Sample Solutions

1. **At the 6^{th} grade school dance, there are 132 boys, 89 girls, and 14 adults.**

   a. **Write the ratio of number of boys to number of girls.**

      $132:89$ *(answers will vary)*

   b. **Write the same ratio using another form (A: B vs. A to B).**

      132 *to* 89 *(answers will vary)*

   c. **Write the ratio of number of boys to number of adults.**

      $132:14$ *(answers will vary)*

   d. **Write the same ratio using another form.**

      132 *to* 14 *(answers will vary)*

2. **In the cafeteria, 100 milk cartons were put out for breakfast. At the end of breakfast, 27 remained.**

   a. **What is the ratio of milk cartons taken to total milk cartons?**

      $73:100$ *(answers will vary)*

   b. **What is the ratio of milk cartons remaining to milk cartons taken?**

      $27:73$ *(answers will vary)*

3. **Choose a situation that could be described by the following ratios, and write a sentence to describe the ratio in the context of the situation you chose.**

For example:

$3:2$ **When making pink paint, the art teacher uses the ratio $3:2$. For every 3 cups of white paint she uses in the mixture, she needs to use 2 cups of red paint.**

a. 1 to 2

For every one cup of water, there are two half cups of water (answers will vary)

b. 29 to 30

For every 29 girls in the cafeteria, there are 30 boys (answers will vary)

c. $52:12$

For every 52 weeks in the year, there are 12 months (answers will vary)

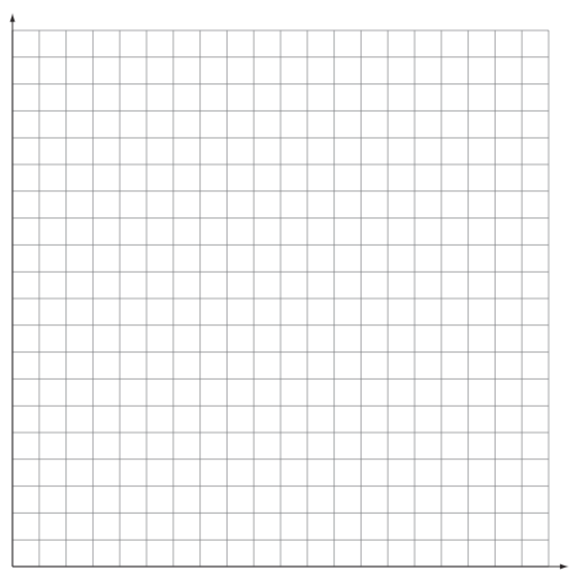
Name ______________________________ Date ________________

1. The most common women's shoe size in the U.S. is reported to be an 8 ½. A shoe store uses a table like the one below to decide how many pairs of size 8 ½ shoes to buy when they place a shoe order from the shoe makers.

Total number of pairs of shoes being ordered	Number of pairs of size 8 ½ to order
50	8
100	16
150	24
200	32

   a. What is the ratio of the number of pairs of size 8 ½ shoes they order to the total number of pairs of shoes being ordered?

   b. Plot the values from the table on a coordinate plane, and draw a straight line through the points. Label the axes. Then use the graph to find the number of pairs of size 8 ½ shoes they order for a total order of 125 pairs of shoes.



2. Wells College in Aurora, New York was previously an all-girls college. In 2005, the college began to allow boys to enroll. By 2012, the ratio of boys to girls was 3 to 7. If there were *200 more girls than boys* in 2012, how many boys were enrolled that year? Use a table, graph, or tape diagram to justify your answer.

3. Most television shows use *13 minutes of every hour* for commercials, leaving the remaining 47 minutes for the actual show. One popular television show wants to change the ratio of commercial time to show time to be *3:7*. Create two ratio tables, one for the normal ratio of commercials to programming and another for the proposed ratio of commercials to programming. Use the ratio tables to make a statement about which ratio would mean fewer commercials for viewers watching 2 hours of television.

A Progression Toward Mastery					
Assessment Task Item		**STEP 1** Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	**STEP 2** Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	**STEP 3** A correct answer with some evidence of reasoning or application of mathematics to solve the problem, <u>or</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	**STEP 4** A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	**a** 6.RP.1 6.RP.3a	The ratio given is incorrect and does not reflect an associated ratio. The student does not display an understanding of determining ratio using a ratio table.	The ratio given is an associated ratio such as 25:4. It may or may not be expressed in the smallest unit possible. There is evidence that the student understands how to determine a ratio from a ratio table, but lacks attentiveness to the precision of which ratio is being asked for.	The ratio given is the correct ratio 4:25, but may be expressed using a larger unit, such as 8:50. The notation or wording of the ratio statement may have minor errors.	The ratio is given correctly as 4:25. The notation and/or wording of the ratio statement are correct.
	b 6.RP.1 6.RP.3a	The student did not produce a graph or the graph does not accurately depict the pairs from the table. The student was unable to answer the question correctly.	The student depicted a graph but the graph contains more than one error in its depiction, such as not going through the given points, not labeling the axes, or not depicting a line through the origin. The student may or may not have answered the question correctly.	The student depicted a graph but the graph contains a minor error in its depiction, such as not accurately plotting the given points, not labeling the axes, <u>OR</u> depicting a line that just misses going through the origin. The student answered the question correctly or incorrectly, but the answer would be correct given their depiction of the graph.	The student depicted the graph correctly, including plotting the given points, labeling the axes, <u>AND</u> depicting a line that goes through the origin. The student answered the question correctly, and the answer is represented in the graph.

2	**6.RP.3 (Stem Only)**	The student was unable to answer the question. They were not able to accurately depict the ratio of boys to girls, or showed no evidence of moving beyond that basic depiction.	The student depicted the ratio of boys to girls, and showed some evidence of using their depiction to solve the problem, but was unable to come to a correct answer. The answer was either incomplete or incorrect.	The student was able to choose a depiction of the ratio and to incorporate the other information given into their depiction, but made an error in arriving at the answer.	The student was able to choose a depiction of the ratio of boys to girls and incorporate into their depiction the additional information of the difference between the number of girls and the number of boys. The student was able to use their depiction to arrive at the correct answer.
3	**6.RP.3a**	The student was not able to complete the two tables OR was not able to fill in at least one row in each table. The student was unable to compose a reasonably accurate comparison of which option would be better for viewers.	The student constructed ratio tables with at least one entry in each table and demonstrated some reasoning in making a statement of comparison, even if the statement did not match the table entries.	The student made two ratio tables with at least two entries in each table. There were one or more errors in the entries of the table. The student was able to make a statement of comparison of which option was better for viewers based on the entries they provided in the table.	The student made two ratio tables with at least two entries in each table. The student was able to make an accurate comparison of which option was better for viewers and relate their comparison to a 2-hour show using accurate grade level language.

Name ______________________________ Date ______________

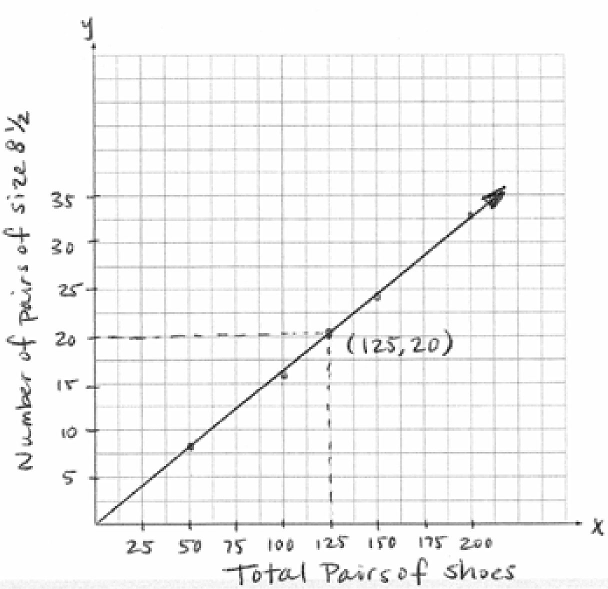
1. The most common women's shoe size in the U.S. is reported to be an 8 ½. A shoe store uses a table like the one below to decide how many pairs of size 8 ½ shoes to buy when they place a shoe order from the shoe makers.

Total number of pairs of shoes being ordered	Number of pairs of size 8 ½ to order
50	8
100	16
150	24
200	32

a. What is the ratio of the number of pairs of size 8 ½ shoes they order to the total number of pairs of shoes being ordered?

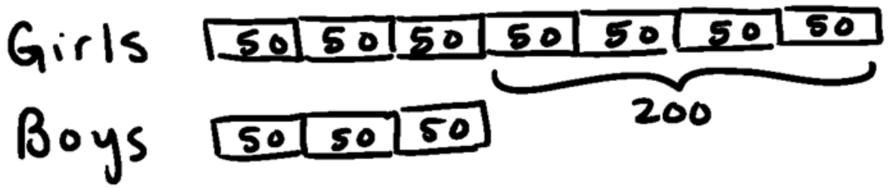
The ratio of size 8½ shoes to total shoes is 4 : 25.

b. Plot the values from the table on a coordinate plane, and draw a straight line through the points. Label the axes. Then use the graph to find the number of pairs of size 8 ½ shoes they order for a total order of 125 pairs of shoes.



They should order 20 pairs of size 8½ shoes if the total order is 125 pairs of shoes.

2. Wells College in Aurora, New York was previously an all-girls college. In 2005, they began to allow boys to enroll at the college. By 2012, the ratio of boys to girls was 3 to 7. If there were *200 more girls than boys* in 2012, how many boys were enrolled that year? Use a table, graph, or tape diagram to justify your answer. (6.RP.3 – Stem Only)



150 boys were enrolled in 2012.

3. Most television shows use *13 minutes of every hour* for commercials, leaving the remaining 47 minutes for the actual show. One popular television show wants to change the ratio of commercial time to show time to be *3:7*. Create two ratio tables, one for the normal ratio of commercials to programming and another for the proposed ratio of commercials to programming. Use the ratio tables to make a statement about which ratio would mean fewer commercials for viewers watching 2 hours of television. (6.RP.3a)

Normal

Total Time	Commercial Time	Show Time
60	13	47
120	26	94

Changed

Total Time	Commercial Time	Show Time
10	3	7
60	18	42
120	36	84

The normal way is better for viewers. In a 2 hour show the normal way uses 26 min for commercials, but the new way would use 36 min. for commercials.

Name ____ Date ______________________

1. Jasmine has taken an online boating safety course and is now completing her end of course exam. As she answers each question, the progress bar at the bottom of the screen shows what portion of the test she has finished. She has just completed question 16 and the progress bar shows she is 20% complete. How many total questions are on the test? Use a table, diagram, or equation to justify your answer.

2. Alisa hopes to play beach volleyball in the Olympics someday. She has convinced her parents to allow her to set up a beach volleyball court in their back yard. A standard beach volleyball court is approximately 26 feet by 52 feet. She figures that she will need the sand to be one foot deep. She goes to the hardware store to shop for sand and sees the following signs on pallets containing bags of sand.

a. What is the rate that Brand A is selling for? Give the rate and then specify the unit rate.

b. Which brand is offering the better value? Explain your answer.

c. Alisa uses her cell phone to search how many pounds of sand is required to fill 1 cubic foot and finds the answer is 100 pounds. Choose one of the brands and compute how much it will cost Alisa to purchase enough sand to fill the court. Identify which brand was chosen as part of your answer.

3. Loren and Julie have different part time jobs after school. They are both paid at a constant rate of dollars per hour. The tables below show Loren and Julie's total income (amount earned) for working a given amount of time.

Loren

Hours	2	4	6	8	10	12	14	16	18
Dollars	18	36	54	72	90	108			162

Julie

Hours	3	6	9	12	15	18	21	24	27
Dollars	36		108	144	180	216		288	324

a. Find the missing values in the two tables above.

b. Who makes more per hour? Justify your answer.

c. Write how much Julie makes as a rate. What is the unit rate?

d. How much money would Julie earn for working 16 hours?

e. What is the ratio between how much Loren makes per hour and how much Julie makes per hour?

f. Julie works $\frac{1}{12}$ hours/dollar. Write a one or two-sentence explanation of what this rate means. Use this rate to find how long it takes for Julie to earn \$228.

4. Your mother takes you to your grandparents' house for dinner. She drives 60 minutes at a constant speed of 40 miles per hour. She reaches the highway and quickly speeds up and drives for another 30 minutes at constant speed of 70 miles per hour.

   a. How far did you and your mother travel altogether?

   b. How long did the trip take?

   c. Your older brother drove to your grandparents' house in a different car, but left from the same location at the same time. If he traveled at a constant speed of 60 miles per hour, explain why he would reach your grandparents house first. Use words, diagrams, or numbers to explain your reasoning.

A Progression Toward Mastery					
Assessment Task Item		**STEP 1** **Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.**	**STEP 2** **Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.**	**STEP 3** **A correct answer with some evidence of reasoning or application of mathematics to solve the problem, <u>or</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.**	**STEP 4** **A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.**
1	**6.RP.3c**	The student was unable to depict the problem using a table, diagram, or equation, and the student either answered incorrectly <u>OR</u> did not answer the question at all.	The student depicted the problem using a table, diagram, or equation, but had significant errors in their reasoning or calculations, leading to an incorrect answer.	The student was able to answer the question correctly, but was not able to explain their reasoning process with an accurate depiction using a table, diagram, or equation. <u>OR</u> The student gave an accurate depiction of the problem, but made a minor calculation or articulation error in arriving at the answer.	The student gave an accurate depiction of the problem with a table, diagram, or equation <u>AND</u> connected that depiction to a correct answer to the question.
2	**a** **6.RP.2** **6.RP.3d**	The student was unable to answer the question. They were not able to accurately represent the rate for Brand A or the unit rate for Brand A. The student showed no evidence of moving beyond that representation.	The student was able to accurately represent the rate for Brand A, but was unable to determine the unit rate. The student is unable to apply the unit rate to further questioning in the problem.	The unit rate is given correctly as 12, but the work lacks connection to the original problem of 60 lb. per $5.	The rate is given correctly as 12 pounds per dollar <u>AND</u> the unit rate is given as 12.

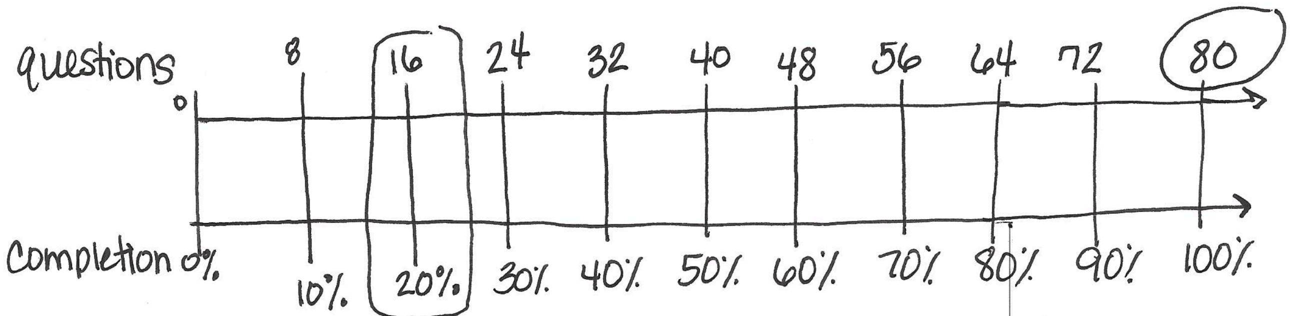
	b **6.RP.2** **6.RP.3d**	The student was unable to answer the question. They were not able to accurately represent the rate or unit rate for Brand B and showed no evidence of moving beyond that representation.	The student was able to accurately represent the rate for Brand B, but was unable to apply the unit rate in comparison to the unit rate of Brand A.	The student accurately represented the unit rate of Brand B as 12.5 lb. per $1 AND compared the unit rate to being more than Brand A. The student did not make connections to the problem and did not determine that Brand B was a better deal because it gives more sand than Brand A.	The student accurately represented both unit rates of Brand A and Brand B. The student determined Brand B was a better unit rate and related the unit rates to the problem.
	c **6.RP.2** **6.RP.3d**	The student did not answer the question correctly. The total number of cubic feet was not found. The rate of 100 lb./1 ft. was not used to determine the total pounds of sand and the unit rate of the cost of either A or B was not used to determine the total cost of the project.	The student determined the total number of cubic feet. The rates to find the total pounds of sand needed were not used or were miscalculated. The unit rate of the cost of A or B was not used to determine the total cost of the project or was miscalculated.	The student accurately determined the number of cubic feet needed for the project. The rate of 100 lb./1 ft. was accurately calculated to determine the total pounds of sand needed. The rate of $1/the unit rate of A or B to determine the final cost was miscalculated.	The student accurately determined the total cubic feet needed, the total pounds of sand needed and used the appropriate rate to determine the final cost of the project. The student used labels accurately to support the reasoning of the final answer.
3	**a** **6.RP.1** **6.RP.2** **6.RP.3a** **6.RP.3b**	The student was unable to answer the question. The values were not placed in either table OR incorrect values were provided.	The student was able to provide two to three correct values to portions of the tables, but did not support the answers mathematically.	The student was able to provide correct values for three to four portions of the tables, but did not support the answers mathematically.	The student was able to provide correct values for all portions of the tables. The student provided reasoning for the answers using additive patterns and unit rate conversion.
	b **6.RP.1** **6.RP.2** **6.RP.3a** **6.RP.3b**	The student did not calculate the hourly rate of either Loren or Julie correctly OR did not answer the question. The rates to determine a final answer were not compared.	The student did not correctly calculate the hourly rate of either Loren or Julie AND was unable to compare the rates and determine which girl made more money per hour.	The student correctly calculated the hourly rate of each girl, but did not compare the rates to determine which made more money per hour.	The student accurately answered the question AND justified their reasoning through comparison of the hourly rates.
	c **6.RP.1** **6.RP.2** **6.RP.3a** **6.RP.3b**	The student was unable to answer the question. The rate or the unit rate was not accurately determined. The student did not make connections to the values in the table.	The student referenced values from the table. (e.g., $36/3 hrs.), but did not express the values as a rate or a unit rate.	The student correctly determined the rate of Julie's pay as $12 for every hour, but did not determine the unit rate to be 12.	The student accurately answered the question by representing the unit rate as 12 AND by referencing the values from the table.

	d **6.RP.1** **6.RP.2** **6.RP.3a** **6.RP.3b**	The student was unable to answer the question. The correct rate with the amount of hours was not accurately computed <u>OR</u> the student did not attempt the problem.	The student did not accurately compute the correct rate with the amount of hours, but was proficient in the process to find the correct answer.	The student computed the correct rate with the amount of hours. The student found the total amount of money Julie made in 16 hours. Student work lacked labeling and clear sequence in solving.	The student accurately derived the correct amount of money Julie made in 16 hours. The student used the correct rate and the work was labeled in order to justify the reasoning. The student's work is in logical progression.
	e **6.RP.1** **6.RP.2** **6.RP.3a** **6.RP.3b**	The student was unable to answer the question. The correct rate of pay for one or both of the girls was not found.	The student was able to compute the accurate rate of pay for the girls, but did not compare to determine which girl made more money per hour.	The student accurately computed the rate of pay for each girl <u>AND</u> accurately compared the pay in ratio form. The student did not derive a simplified ratio from the rates of pay.	The student answered the problem accurately, with labels and simplified their final answer.
	f **6.RP.1** **6.RP.2** **6.RP.3a** **6.RP.3b**	The student explained what the rate meant in the problem, but did not accurately find the answer.	The student explained the meaning of the rate in detail using conversions, but made errors when deriving the plan to solve. *Example: The answer is not indicative of understanding cancellation of units and finds $19 instead of 19 hours.*	The student's explanation was lucid with conversions and support. The student may have multiplied by minute conversion and found a final answer of 1,140 minutes instead of 19 hours.	The student answered the problem with precision and with coherent explanation of what the rate means. Calculations are accurate and the final answer is supported and justified through appropriate labeling.
4	**a** **6.RP.3b**	The student was unable to answer the problem accurately. They student was not able to apply the rates to determine the amount of miles.	The student was able to show their intent to multiply the rate by the time to find the miles, but computed incorrectly.	The student multiplied the rates appropriately to the time for each section of the trip. The amount of separate miles was found, but the student did not combine them for a total amount of miles for the trip. <u>OR</u> The student showed understanding of the concept, but made computation errors.	The student completed the entire problem accurately with appropriate labels. The student was able to derive a total distance with no computation errors.

	b **6.RP.3b**	The student did not complete the problem OR answered with an incorrect response.	The student used information from the original problem to determine the addends, but computed the total incorrectly.	The student used information from the original problem to determine addends AND computed the sum correctly, but did not report the correct unit.	The student used information from the original problem to determine addends AND computed the sum correctly. The student labeled work appropriately and converted the minutes into hours.
	c **6.RP.3b**	The student did not use a diagram, words, or numbers to support the answer OR used the diagram inappropriately. The student did not answer the problem with an accurate response.	The student provided an accurate response, but did not utilize a diagram, words, or numbers to support the answer.	The student provided a correct answer and used only words or numbers to support the answer.	The student used appropriate diagrams, words, and numbers to support the accurate answer.

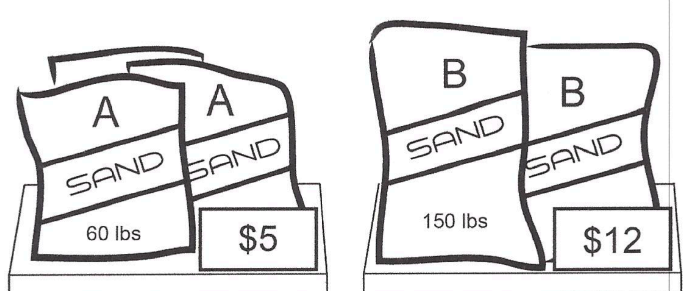
Name ____________________ Date ____________________

1. Jasmine has taken an online boating safety course and is now completing her end of course exam. As she answers each question, the progress bar at the bottom of the screen shows what portion of the test she has finished. She has just completed question 16 and the progress bar shows she is 20% complete. How many total questions are on the test? Use a table, diagram, or equation to justify your answer. (6.RP.3c)



There are 80 questions on the test.

2. Alisa hopes to play beach volleyball in the Olympics someday. She has convinced her parents to allow her to set up a beach volleyball court in their back yard. A standard beach volleyball court is approximately 26 feet by 52 feet. She figures that she will need the sand to be one foot deep. She goes to the hardware store to shop for sand and sees the following signs on pallets containing bags of sand. (6.RP.2, 6.RP.3d)



a. What is the rate that Brand A is selling for? Give the rate and then specify the unit rate.

Brand A → $\dfrac{60 \text{ lbs.} \div 5}{\$5 \div 5} = \dfrac{12 \text{ lbs.}}{\$1} = 12$

rate   unit rate

b. Which brand is offering the better value? Explain your answer.

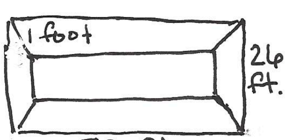
Brand B → $\dfrac{150 \text{ lbs.}}{\$12}$

$\dfrac{150 \text{ lbs} \div 12}{\$12 \div 12} = \dfrac{12.5 \text{ lbs}}{\$1}$

Brand A is selling sand at a rate of 12 lbs per dollar. Brand B is selling at a rate of 12.5 lbs. per dollar. Brand B offers a better value because it gives more sand per dollar.

c. Alisa uses her cell phone to search how many pounds of sand is required to fill 1 cubic foot and finds the answer is 100 pounds. Choose one of the brands and compute how much it will cost Alisa to purchase enough sand to fill the court. Identify which brand was chosen as part of your answer.

Choose Brand A.



$52 \text{ ft.} \times 26 \text{ ft.} \times 1 \text{ ft.} = 1{,}352 \text{ ft.}^3$

$1{,}352 \cancel{\text{ft}}^3 \times \dfrac{100 \text{ lbs}}{1 \cancel{\text{ft.}}^3} =$

135,200 lbs.

$135{,}200 \cancel{\text{lbs}} \times \dfrac{\$1}{12 \cancel{\text{lbs.}}} \approx \$11{,}266.67$

Alisa will need \$11,266.67

Choose Brand B.

$52 \text{ ft.} \times 26 \text{ ft.} \times 1 \text{ ft.} = 1{,}352 \text{ ft.}^3$

$1{,}352 \cancel{\text{ft.}}^3 \times \dfrac{100 \text{ lbs.}}{1 \cancel{\text{ft}}^3} = 135{,}200 \text{ lbs.}$

$135{,}200 \cancel{\text{lbs.}} \times \dfrac{\$1}{12.5 \cancel{\text{lbs}}} = \$10{,}816$

Alisa will need \$10,816

3. Loren and Julie have different part time jobs after school. They are both paid at a constant rate of dollars per hour. The tables below show Loren and Julie's total income (amount earned) for working a given amount of time. (6.RP.1, 6.RP.2, 6.RP.3a, 6.RP.3b)

Loren

+2 +2 +2 +2

Hours	2	4	6	8	10	12	14	16	18
Dollars	18	36	54	72	90	108	126	144	162

+18 +18 +18 +18

$$\begin{array}{r} 108 \\ +\ 18 \\ \hline 126 \end{array} \qquad \begin{array}{r} 126 \\ +\ 18 \\ \hline 144 \end{array}$$

Julie

+3 +3 +3

Hours	3	6	9	12	15	18	21	24	27
Dollars	36	72	108	144	180	216	252	288	324

+36 +36 +36 +36

a. Find the missing values in the two tables above.

$$\begin{array}{r} 216 \\ +\ 36 \\ \hline 252 \end{array} \qquad \begin{array}{r} 252 \\ +\ 36 \\ \hline 288 \end{array} \checkmark \qquad \begin{array}{r} 36 \\ +\ 36 \\ \hline 72 \end{array}$$

ratio 3:36 = 1:12
so, 6:72

b. Who makes more *per hour*? Justify your answer.

Loren 2 hrs : \$18 → 1 hr. : \$9
Julie 3 hrs : \$36 → 1 hr. : \$12

Loren Julie
\$9 < \$12

Julie makes more per hour

c. Write how much Julie makes as a rate. What is the unit rate?

Julie 3 : 36 → 1 : 12

\$12 per hour
\$12/hour
unit rate 12

d. How much money would Julie earn for working 16 hours?

Julie's rate $\frac{\$12}{1 \text{ hour}} \times 16 \text{ hours} = \frac{\$12 \times 16 \cancel{\text{hrs}}}{1 \cancel{\text{hrs}}} =$

$\$12 \times 16 = \$192$

e. What is the ratio between how much Loren makes per hour and how much Julie makes *per hour*?

Loren : Julie

$9/hr : $12/hr

9 : 12

↓

3 : 4

f. Julie works $\frac{1}{12}$ hours/dollar. Write a one or two-sentence explanation of what this rate means. Use this rate to find how long it takes for Julie to earn $228.

To earn one dollar, Julie has to work $\frac{1}{12}$ of an hour (or 5 minutes).

$\frac{\frac{1}{12} \text{ hrs.}}{\$1} \times \$228 = \frac{\frac{1}{12} \text{ hrs.} \times \cancel{\$}228}{\cancel{\$}1} = \frac{1}{12} \text{ hrs.} \times 228 = 19 \text{ hrs}$

$$\begin{array}{r} 19 \\ 12\overline{)228} \\ -12 \\ \hline 108 \\ -108 \\ \hline 0 \end{array}$$

4. Your mother takes you to your grandparents' house for dinner. She drives 60 minutes at a constant speed of 40 miles per hour. She reaches the highway and quickly speeds up and drives for another 30 minutes at constant speed of 70 miles per hour. (6.RP.3b)

a. How far did you and your mother travel altogether?

$1\cancel{hr} \times \frac{40 mi}{\cancel{hr}} = 40$ miles

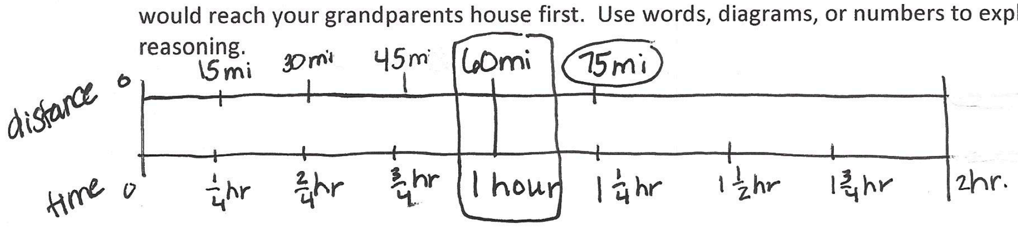
$0.5\cancel{hr} \times \frac{70 mi}{\cancel{hr}} = 35$ miles

40 miles + 35 miles = 75 miles

b. How long did the trip take?

60 minutes + 30 minutes = 90 minutes or $1\frac{1}{2}$ hours.

c. Your older brother drove to your grandparents' house in a different car, but left from the same location at the same time. If he traveled at a constant speed of 60 miles per hour, explain why he would reach your grandparents house first. Use words, diagrams, or numbers to explain your reasoning.



The trip is 75 miles long. If he travels 60 miles in 1 hour, it will take him $1\frac{1}{4}$ or 1.25 hours to get there.