High School Math

Grades 9–11

Scope and Sequence

TRADITIONAL

Grade 9 Algebra One

A0	Introductory Unit	5 days	5
A1	Modeling with Functions	15 days	20
A2	Linear Functions	15 days	35
A3	Linear Equations and Inequalities in One Variable	15 days	50
M1	Modeling Unit	4 days	54
A4	Linear Equations and inequalities in Two Variables	15 days	69
P1	Project	5 days	74
A5	Quadratic Functions	20 days	94
A6	Quadratic Equations	26 days	120
M2	Modeling Unit	4 days	124
S1	Statistics	30 days	154
P2	Project	5 days	159
Total		159	

Grade 10 Geometry

G0	Introduction and Construction Unit	12 days	12
G1	Basic Definitions and Rigid Motions	20 days	32
G2	Geometric Relationships and Properties	15 days	47
G3	Similarity	20 days	67
M3	Modeling Unit	4 days	71
P3	Project	5 days	76
G4	Coordinate Geometry	15 days	91
G5	Circles and Conics	15 days	106
G7	Geometric Measurement and Dimensions	15 days	121
G6	Trigonometric Ratios	15 days	136
M4	Capstone Geometric Modeling Project	10 days	146
P4	Project	5 days	151
Total		156	

Grade 11 Algebra Two

A0	Introductory Unit	5 days	5
A7	Exponential Functions	25 days	30
A8	Trigonometric Functions	15 days	45
A9	Functions	15 days	60
M5	Modeling Unit	8 days	68
P5	Project	5 days	73
A10	Rational and Polynomial Expressions	20 days	93
P1	Probability	28 days	121
S2	Statistics (Random Processes)	26 days	147
M6	Modeling Unit	4 days	151
P6	Project	5 days	156
Total		156	

INTEGRATED

Grade 9

I-0	Introductory Unit	5 days	5
A1	Modeling with Functions	15 days	20
A2	Linear Functions	15 days	35
A3	Linear Equations and Inequalities in One Variable	15 days	50
A4	Linear Equations and Inequalities in Two variables	15 days	65
M1	Modeling Unit	4 days	69
P1	Project	5 days	74
G0	Tools and Construction Unit	10 days	84
G1	Basic Definitions and Rigid Motions	20 days	104
G2	Geometric Relationships and Properties	15 days	119
M2	Modeling Unit	4 days	123
S1	Statistics	30 days	153
P2	Project	5 days	158
Total		158	

Grade 10

I-0	Introductory Unit	5 days	5
G4	Coordinate Geometry	15 days	20
A5	Quadratic Functions	20 days	40
A6	Quadratic Equations	26 days	66
M3	Modeling Unit	4 days	70
P3	Project	5 days	75
G3	Similarity	20 days	95
G5	Circles and Conics	20 days	115
P1	Probability	28 days	143
M4	Capstone Geometric Modeling Project	10 days	153
P4	Project	5 days	158
Total		158	

Grade 11

I-0	Introductory Unit	5 days	5
G7	Geometric Measurement and Dimension	15 days	20
A7	Exponential Functions	25 days	45
G6	Trigonometric Ratios	15 days	60
M5	Modeling Unit	8 days	68
P5	Project	5 days	73
A8	Trigonometric Functions	15 days	88
A9	Functions	15 days	103
A10	Rational and Polynomial Expressions	20 days	123
S2	Statistics (Random Processes)	26 days	150
M6	Modeling Unit	4 days	154
P6	Project	5 days	149
Total		158	

High School Algebra Units:

High School Geometry Units:

High School Statistics and Probability Units:

High School Modeling Units:

M1 Modeling

M2 Modeling

M3 Modeling

M4 Capstone Geometric Modeling Project (10 days)

M5 Modeling (8 days)

M6 Modeling

Algebra

A0   Introductory Unit (5 days)

This unit will review expressions and manipulations of expression.

Algebra

A1   Modeling with Functions (15 days)

Description

This initial unit starts with a treatment of quantities as preparation for work with modeling. The work then shifts to a general look at functions with an emphasis on representation in graphs, and interpretation of graphs in terms of a context. More emphasis is placed on qualitative analyses than calculation and symbolic manipulation. Linear and non-linear examples are explored.

Quantities
A short treatment of the general notion of a "quantity" thought of as a number with a specific unit of measure. Includes unit analysis (dimensional analysis).

Examples of simple quantities with standard units of measure; the fundamental dimensions of quantities (length, time, weight, temperature, counts); division of quantities: quotient units; examples of quantities with quotient units: speed, flow rate, frequency, price, density, pressure; quotient units and "rates"; quotient units and unit conversion; unit analysis/dimensional analysis; multiplication of quantities: product units; area and volume as examples of quantities with product units; person-days and kilowatt hours as other examples of product units;

Functions
A general treatment of the function concept with minimal use of symbolic expressions, and instead emphasis on the idea of a function as a mapping represented in graphs or tables. The functions used in this unit, will be mostly linear and 'baby exponential'. In grade 11, student will thoroughly study exponential functions. But they will be introduced to them here so they can compare two different types of functions. Quadratics or piecewise functions can be used to illustrates the properties of functions.

Domain and range; functions defined by graphs and their interpretation; functions defined by tables and their interpretation; properties of particular functions (rate of change, zeros) and their meaning in an application; sums and differences of two functions; product of a function and a constant; vertical shifts and horizontal shifts; equality of two functions vs. values where two functions are equal; equations defined in terms of functions and their solution; functions defined by geometric conditions (projections); functions defined recursively; sequences.

This unit builds on 8.F 1, 8.F 2, 8.F 3
Functions: Define, evaluate, and compare functions.
and 8.F 4 and 8.F 5
Functions: Use functions to model relationships between quantities.

Algebra

Quantities: Subunit 1

Quantities   N-Q
Reason quantitatively and use units to solve problems.
N-Q 1 (page 60)
Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N-Q 2 (page 60)
Define appropriate quantities for the purpose of descriptive modeling.

N-Q 3 (page 60)
Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Functions: Subunit 2

Interpreting Functions   F-IF
Understand the concept of a function and use function notation.
F-IF 1 (page 69)
Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F-IF 2 (page 69)
Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Understand the concept of a function and use function notation.
F-IF 3 (page 69)
Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1*

Interpret functions that arise in applications in terms of the context.
F-IF 4 (page 69)
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

F-IF 5 (page 69)
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

Algebra

Interpreting Functions   F-IF
Analyze functions using different representations.
F-IF 9 (page 70)
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Linear, Quadratic, and Exponential Models*   F-LE
(Only linear, simple quadratic, and simple exponential functions.)
Construct and compare linear, quadratic, and exponential models and solve problems.
F-LE 1 (page 70)
Distinguish between situations that can be modeled with linear functions and with exponential functions.

F-LE 1a (page 70)
Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

F-LE 1b (page 70)
Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

F-LE 3 (page 71)
Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Interpret expressions for functions in terms of the situation they model.
F-LE 5 (page 71)
Interpret the parameters in a linear or exponential function in terms of a context.

Algebra

A2 Linear Functions (15 days)

Description

A thorough treatment of linear functions of one variable $f(x) = y_0 + kx$.

Contents:
Representation of linear functions using expressions, graphs, and tables; identifying and interpreting the three parameters x-intercept x_0, y-intercept y_0, and rate of change k; creating expressions for linear functions using each pair of parameters ($y = y_0 + kx$, $y = k(x - x_0)$, $(x/x_0) + (y/y_0) = 1$); understanding geometrically why the graph is a line; seeing uniform change as the unique feature of linear functions; modeling a variety of situations using functions; working at in-depth solutions of selected problems; looking at the special properties of pure linear functions $y = ax$ and their role in representing proportional relationships; linear sequences ("arithmetic" sequences); working with the absolute value function $f(x) = |y_0 + kx|$.

> This unit builds on 8.EE 5 and 8.EE 6
> *Expressions and Equations: Understand the connections between proportional relationships, lines, and linear equations.*
> and 8.F 2 and 8.F 3
> *Functions: Define, evaluate, and compare functions.*
> *Use functions to model relationships between quantities.*

Interpreting Functions   F-IF
Interpret functions that arise in applications in terms of the context.
F-IF 6 (Page 69) Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
(Just introduce concept — will be covered again in quadratics and in other functions units.)

Interpreting Functions   F-IF
Analyze functions using different representations.
F-IF 7 (page 69)
Graph functions expressed symbolically and show key features of
the graph, by hand in simple cases and using technology for more
complicated cases.

F-IF 7a (page 69)
Graph linear functions and show intercepts, maxima, and minima.

F-IF 9 (page 70)
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). ~~For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum~~ (Only linear functions.)

Building functions   F-BF
Build a function that models a relationship between two quantities.
F-BF 1 (page 70)
Write a function that describes a relationship between two quantities.* (Only linear functions.)

F-BF 1a (page 70)
Determine an explicit expression, a recursive process, or steps for calculation from a context.

Algebra

Build a function that models a relationship between two quantities (Only arithmetic sequences.)
F-BF 2 (page 70)

Write arithmetic ~~and geometric~~ sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Build new functions from existing functions
F-BF 4 (page 70)

Find inverse functions.

F-BF 4a (page 70) (Only linear functions.)

Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x^3 or f(x) = (x+1)/(x–1) for x ≠ 1*

Linear, Quadratic, and Exponential Models*   F-LE
Construct and compare linear, quadratic, and exponential models
and solve problems
F-LE 1 (page 70)
Distinguish between situations that can be modeled with linear functions and with exponential functions.

F-LE 1a (page 70)
Prove that linear functions grow by equal differences over equal intervals, ~~and that exponential functions grow by equal factors over equal intervals.~~

F-LE 1b (page 70)
Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. (Only linear functions.)

Algebra

A3   Linear Equations and Inequalities in One Variable (15 days)

Description

A thorough treatment to solution of linear equations in one unknown $ax + b = cx + d$, with an extension to solution of linear inequalities $ax + b < c$.

Solving Linear Equations and Inequalities:
Connecting linear equations to linear functions; solving linear equations both through manipulation of expressions and graphically; understanding the conditions under which a linear equation has no solution, one solution, or an infinite number of solutions; seeing the solution of a linear equation $ax + b = cx + d$ as involving the intersection of the graphs of two linear functions; also seeing the solution x as the number where two linear functions have the same value; solving a linear equation step by step and graphing each step; reducing any linear equation to the normal form $Ax + B = 0$, and finding the solution $x = -(B/A)$; solving linear inequalities $ax + b < c$ and $|ax + b| < c$, and representing the solution on a number line;

Creating Linear Equations:
Solving a wide variety of problems in applied settings; seeing that in order to obtain an equation we have to express the same quantity in two different ways; solving a linear equation in an applied setting to get a number, then replacing one of the numerical parameters in the setting with a parameter, and solving again, this time getting a function of the parameter.

> This unit builds on 8.EE 7
> *Expressions and Equations: Analyze and solve linear equations and pairs of simultaneous linear equations.*

Solving Linear Equations and Inequalities: Subunit 1

Reasoning with Equations and Inequalities   A-REI
Understand solving equations as a process of reasoning and explain the reasoning.
A-REI 1 (page 65)
Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Reasoning with Equations and Inequalities   A-REI
Solve equations and inequalities in one variable.
A-REI 3 (page 65)
Solve linear equations and inequalities and inequalities in one variable, including equations with coefficients represented by letters.

Reasoning with Equations and Inequalities   A-REI
Represent and solve equations and inequalities graphically.
A-REI 11 (page 66)
Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

Algebra

Creating Linear Equations: Subunit 2

Creating equations*   A-CED
Create equations that describe numbers or relationships.
A-CED 1 (page 65)
Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

A-CED 3 (page 65)
Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A-CED 4 (page 65)
Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.

Algebra

A4　Linear Equations and Inequalities in Two Variables (15 days)

A thorough treatment of linear equations in two unknowns $ax + by + c = 0$, including simultaneous solution of two such equations.

Seeing the differences among the solution to a single equation in one unknown, solutions to a single equation in two unknowns, and "simultaneous" solutions to two equations in two unknowns; explore briefly linear functions in two variables $F(x,y) = ax + by + c$ and the three-dimensional graph (a plane) $z = ax + by + c$ of such functions; see the solution to an equation $ax + by + c = 0$ in two unknowns as the intersection of the graph of this function with the x-y plane; make the analogy with functions and equations of one variable/unknown; explore the symmetric form $(x/x_0) + (y/y_0) = 1$ of a linear equation in two unknowns and compare equations of conic sections; see the solution of any equation in two unknowns as a line or curve in the x-y plane; find simultaneous solutions to pairs of equations $ax + by + c = 0$ and $a'x + b'y + c' = 0$ symbolically; solve many problems set in an applied context using such equations.

Creating equations*　　　　　　A-CED
Create equations that describe numbers or relationships
A-CED 2 (page 65)
Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED 3 (page 65)
Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A-CED 4 (page 65)
Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.

Reasoning with Equations and Inequalities　　　　　　A-REI
Solve systems of equations.
A-REI 5 (page 65)
Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI 6 (page 66)
Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Reasoning with Equations and Inequalities　　　　　　A-REI
Represent and solve equations and inequalities graphically.
A-REI 10　(page 66)
Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line

A-REI 12 (page 66)
Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Algebra

A5 Quadratic Functions (20 days)

Description

A treatment of quadratic functions in one variable $f(x) = ax^2 + bx + c$ and their applications.

See that the product of two linear expressions $(mx + n)$ and $(px + q)$ leads to a quadratic function of the form $f(x) = ax^2 + bx + c$; represent quadratic functions symbolically and graphically; know that the graph of a quadratic function is a parabola; see the effect of each of the parameters a, b, and c on the graph of $ax^2 + bx + c$; express the vertex of the parabolic graph in terms of these parameters; see the graph of any quadratic function $ax^2 + bx + c$ is a scaled and shifted version of the parabola $y = x^2$; solve max-min problems involving quadratic functions; explore applications of quadratic functions to problems involving area, motion under gravity, stopping distance of a car, and revenue in terms of demand and price.

Interpreting Functions   F-IF
Interpret functions that arise in applications in terms of the context.
F-IF 4 (page 69)
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

F-IF 5 (page 69)
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

F-IF 6 (page 69)
Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

Interpreting Functions   F-IF
Analyze functions using different representations.
F-IF 7 (page 69)
Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

F-IF 7a (page 69)
Graph quadratic functions and show intercepts, maxima, and minima.

F-IF 8 (page 69)
Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-IF 8a (page 69)
Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Algebra

F-IF 9 (page 70)
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building functions   F-BF
Build a function that models a relationship between two quantities.
F-BF 1 (page 70)
Write a function that describes a relationship between two quantities.*

F-BF 1a (page 70)
Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-BF 1b (page 70)
Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Build new functions from existing functions.
F-BF 3 (page 70)
Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them

Algebra

A6 Quadratic Equations (26 days)

A treatment of the solution of quadratic equations in one unknown $ax^2 + bx + c = 0$ and applications.

Explore the solution of the special cases $ax^2 + c = 0$ and $ax^2 + bx = 0$ of quadratic equations both graphically and through symbol manipulation; explore the square root functions $y = \sqrt{ax}$, $y = \sqrt{ax + b}$, $y = \sqrt{ax} + b$; see that the product of two linear expressions $(mx + n)$ and $(px + q)$ leads to a quadratic function of the form $f(x) = ax^2 + bx + c$ and that any quadratic expression $ax^2 + bx + c$ can be factored into a product of two such linear expressions; solve quadratic equations $ax^2 + bx + c = 0$ by completing the square and by

factoring; derive the quadratic formula x $= \dfrac{-b}{2a} \pm \dfrac{\sqrt{b^2 - 4ac}}{2a}$.

Solve quadratic equations in applied contexts; work with a variety of problems, including max min- problems, that involve setting up and solving a quadratic equation.

The Complex Number System   N-CN
Use complex numbers in polynomial identities and equations.
N-CN 7 (page 60)
Solve quadratic equations with real coefficients that have complex solutions.

Seeing Structure in Expressions   A-SSE
Write expressions in equivalent forms to solve problems.
A-SSE 3 (page 64)
Choose and produce an equivalent form of an expression to reveal and
explain properties of the quantity represented by the expression.

A-SSE 3a (page 64)
Factor a quadratic expression to reveal the zeros of the function it defines.

A-SSE 3b (page 64)
Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Reasoning with Equations and Inequalities   A-REI
Solve equations and inequalities in one variable.
A-REI 4 (page 65) Solve quadratic equations in one variable.

A-REI 4a (page 65)
Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

A-REI 4b (page 65)
Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Reasoning with Equations and Inequalities   A-REI
Solve systems of equations.

Algebra

Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y = −3x and the circle $x^2 + y^2 = 3$.*

A7   Exponential Functions (25 days)

Description

A treatment of exponential functions $E(x) = y_0 b^{kx}$.

Laws of exponents: definition of exponent notation; sum law for exponents; product law for exponents; definition of negative exponent notation; value of b^0; definition of unit fractional exponent notation; definition of fractional exponent notation; distributive law for exponent notation;

basic characteristics of exponential functions; symbolic representation y $= E(x) = y_0 b^{kx}$
computing exponential functions; parallels to linear functions; growth and decay; repeated multiplication as the big idea; geometric sequences; geometry of repeated multiplication; recursive definitions;

the meaning of the dependent variable; the meaning of the independent variable; parameters and their meanings; the parameter y_0; the parameter b; the parameter k;

ways of measuring amount of growth (difference, ratio); proportional change; measuring amount of growth of exponential functions; ways of measuring rate of growth; measuring growth rate of exponential functions; compound interest; continuous growth; polynomial approximations.

The Real Number System   N-RN
Extend the properties of exponents to rational exponents.

Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $5^{1/3})^3$ must equal 5.*

Rewrite expressions involving radicals and rational exponents using the properties of exponents

Seeing Structure in Expressions   A-SSE
Write expressions in equivalent forms to solve problems.

Choose and produce an equivalent form of an expression to reveal and

explain properties of the quantity represented by the expression.

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15t *can be rewritten as* (1.151/12)12$t \approx$ 1.01212t *to reveal the approximate equivalent* monthly interest rate if the annual rate is 15%.

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Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.*

Create equations that describe numbers or relationships
A-CED 1 (page 65)
1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from ~~linear and quadratic functions, and simple rational and~~ exponential functions.

Reasoning with Equations and Inequalities   A-REI
Represent and solve equations and inequalities graphically.
A-REI 11 (page 66)
Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

Interpreting Functions   F-IF
Understand the concept of a function and use function notation.
F-IF 3 (page 69)
Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1*

Interpreting Functions   F-IF
Analyze functions using different representations.
F-IF 7 (page 69)
Graph functions expressed symbolically and show key features of
the graph, by hand in simple cases and using technology for more
complicated cases.*

F-IF 7e (page 69)
 e. Graph exponential and logarithmic functions, showing intercepts and end behavior, ~~and trigonometric functions, showing period, midline, and amplitude.~~

Interpreting Functions   F-IF
Analyze functions using different representations.
F-IF 8 (page 69)
Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-IF 8b (page 69)
Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

Building functions   F-BF
Build a function that models a relationship between two quantities
F-BF 1 (page 70)
Write a function that describes a relationship between two quantities.

F-BF 1b (page 70)
Combine standard function types using arithmetic operations. For example, build a function that

Algebra

models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

F-BF 2 (page 70)
Write ~~arithmetic and~~ geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Building functions   F-BF
Build new functions from existing functions.
F-BF 4 (page 70)
Find inverse functions.

F-BF 4a (page 70)
Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x^3 or f(x) = (x+1)/(x−1) for x ≠ 1.*

Linear, Quadratic, and Exponential Models*   F-LE
Construct and compare linear, quadratic, and exponential models and solve problems.
F-LE 1 (page 70)
Distinguish between situations that can be modeled with linear functions and with exponential functions.

F-LE 1a (page 70)
Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

F-LE 1b (page 70)
Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

F-LE 1c (page 70)
Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE 2 (page 71)
Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F-LE 3 (page 71)
Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F-LE 4 (page 71)
For exponential models, express as a logarithm the solution to ab^{ct} where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model.
F-LE 5 (page 71)
Interpret the parameters in ~~a linear or~~ an exponential function in terms of a context.

Algebra

A8   Trigonometric Functions (15 days)

Description

Trigonometric ratios can be thought of as functions of the angles. With the help of the unit circle, the angles do not need to be between 0 and 90 degrees. By extending the domains to all real numbers, these trigonometric functions are used to model circular and periodic motions.

Interpreting Functions   F-IF
Analyze functions using different representations.
F-IF 7 (page 69)
Graph functions expressed symbolically and show key features of
the graph, by hand in simple cases and using technology for more
complicated cases.

F-IF 7e (page 69)
Graph ~~exponential and logarithmic functions, showing intercepts and end behavior, and~~
trigonometric functions, showing period, midline, and amplitude.

Trigonometric Functions   F-TF
Extend the domain of trigonometric functions using the unit circle.
F-TF 1 (page 71)
Understand radian measure of an angle as the length of the arc on the unit circle subtended by
the angle.

F-TF 2 (page 71)
Explain how the unit circle in the coordinate plane enables the extension of trigonometric
functions to all real numbers, interpreted as radian measures of angles traversed
counterclockwise around the unit circle.

Trigonometric Functions   F-TF
Model periodic phenomena with trigonometric functions.
F-TF 5 (page 71)
Choose trigonometric functions to model periodic phenomena with specified amplitude,
frequency, and midline.*

Trigonometric Functions   F-TF
Prove and apply trigonometric identities.
F-TF 8 (page 71)
Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$
given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

Algebra

A9 Functions (15 days)

<table>
<tr><td>Description</td><td>

Contents:

Inverse functions; inverse functions and solving equations; logarithms; logarithmic functions as inverses of exponential functions; logarithmic scales; semi-log plots; the role of the numbers 0 and 1; the laws of exponents and logarithms; polynomial approximations; applications.

</td></tr>
</table>

The Real Number System   N-RN
Extend the properties of exponents to rational exponents.

N-RN 1 (page 60)
Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{\frac{1}{3}}$ to be the cube root of 5 because we want $(5^{\frac{1}{3}})^3 = 5^{(\frac{1}{3})3}$ to hold, so $5^{1/3})^3$ must equal 5.*

N-RN 2 (page 60)
Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Seeing Structure in Expressions   A-SSE
Interpret the structure of expressions.
A-SSE 1 (page 64)
Interpret expressions that represent a quantity in terms of its context.*

A-SSE 1a (page 64)
Interpret parts of an expression, such as terms, factors, and coefficients.

A-SSE 1b (page 64)
Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

Reasoning with Equations and Inequalities   A-REI
Represent and solve equations and inequalities graphically.
A-REI 11 (page 66)
Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

Interpreting Functions   F-IF
Interpret functions that arise in applications in terms of the context.
F-IF 4 (page 69)
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

Algebra

Interpreting Functions   F-IF
Analyze functions using different expressions

F-IF 7 (page 69)
Graph functions expressed symbolically and show key features of
the graph, by hand in simple cases and using technology for more
complicated cases

F-IF 7b (page 69)
Graph square root, cube root, and piecewise-defined functions, including step functions and
absolute value functions.

F-IF 7c (page 69)

Graph polynomial functions, identifying zeros when suitable factorizations are available, and
showing end behavior.

Building functions   F-BF
Build a function that models a relationship between two quantities.
F-BF 1 (page 70)
Write a function that describes a relationship between two quantities.*

F-BF 1a (page 70)
Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-BF 1b (page 70)
Combine standard function types using arithmetic operations. For example, build a function that
models the temperature of a cooling body by adding a constant function to a decaying
exponential, and relate these functions to the model.

Building Functions   F-BF
Build new functions from existing functions.
F-BF 3 (page 70)
Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific
values of k (both positive and negative); find the value of k given the graphs. Experiment with
cases and illustrate an explanation of the effects on the graph using technology. Include
recognizing even and odd functions from their graphs and algebraic expressions for them

Linear, Quadratic, and Exponential Models*   F-LE
Construct and compare linear, quadratic, and exponential models and solve problems
F-LE 3 (page 71)
Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a
quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Algebra

A10 Rational and Polynomial Expressions (20 days)

Description

This unit focus on performing operations on rational and polynomial expressions and simplifying them. In order to understand the rational expressions, students review the real number system.

The Real Number System   N-RN
Use properties of rational and irrational numbers.
N-RN 3 (page 60)
Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

The Complex Number System   N-CN
Perform arithmetic operations with complex numbers.
N-CN 1 (page 60)
Know there is a complex number i such that $i^2=-1$, and every complex number has the form $a + bi$ with a and b real.

N-CN 2 (page 60)
Use the relation $i^2=-1$, and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

The Complex Number System   N-CN
Use complex numbers in polynomial identities and equations.
N-CN 7 (page 60)
Solve quadratic equations with real coefficients that have complex solutions.

Seeing Structure in Expressions   A-SSE
Interpret the structure of expressions.
A-SSE 1 (page 64)
Interpret expressions that represent a quantity in terms of its context.*

A-SSE 1a (page 64)
Interpret parts of an expression, such as terms, factors, and coefficients.

A-SSE 1b (page 64)
Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

A-SSE 2 (page 64)
Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Algebra

Arithmetic with Polynomials and Rational Expressions   A-APR

Perform arithmetic operations on polynomials

A-APR 1 (page 64)

Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials.

A-APR 2 (page 64)

Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

A-APR 3 (page 64)

Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Arithmetic with Polynomials and Rational Expressions   A-APR

Use polynomial identities to solve problems.

A-APR 4 (page 64)

Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*

Arithmetic with Polynomials and Rational Expressions   A-APR

Rewrite rational expressions.

A-APR 6 (page 65)

Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

Reasoning with Equations and Inequalities   A-REI

Understand solving equations as a process of reasoning and explain the reasoning.

A-REI 2 (page 65)

Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Reasoning with Equations and Inequalities   A-REI

Represent and solve equations and inequalities graphically.

A-REI 11 (page 66)

Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

Geometry

G0   Introductory Unit: Tools of Geometry (10 days)

Description

Students explore geometric ideas using key tools of geometry. This unit focuses on classical Euclidean compass and straightedge constructions, both by hand and in dynamic geometry environments. The goal is to both create constructions and explore why they work. The unit then considers other tools, such as string, paper folding, etc. As with the compass and straightedge constructions, it is important to both mechanically use these tools and to work toward mathematical explanations and justifications.

In dynamic geometry environments, a key distinction is between drawing and constructing figure with particular characteristics. For example, if a rectangle is merely drawn to look like a rectangle, then it is easy to mess up the figure by dragging parts of the diagram. But a rectangle that is "constructed" as a rectangle will remain rectangular even when parts of the figure are dragged about. Following Cuoco, we call these "unmessupable" figures.

Congruence   G-CO
Make geometric constructions.
G-CO 12 (page 76)
Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment and constructing a line parallel to a given line through a point not on the line.*

G- CO 13 (page 76)
Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Geometry

G1　　Basic Definitions and Rigid Motions (20 days)

Description

Building upon the informal experiences with basic geometric objects and relationships, the goal is to start increasing the precision of the definitions. The emphasis should be on the role of definitions and communicating mathematical explanations and arguments rather than on developing a deductive axiomatic system. In Grade 8 rigid motions were explored, here they are defined more precisely and their properties explored.

Basic Geometric Definitions: Subunit 1.1

Congruence　　　G-CO
Experiment with transformations in the plane.
G-CO 1 (page 76)
Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G-CO 3 (page 76)
Given of a rectangle, parallelogram, trapezoid, or regular polygon, and describe the rotations and reflections that carry it onto itself.

Rigid Motions: Subunit 1.2

Congruence　　　G-CO
Experiment with transformations in the plane.
G-CO 2 (page 76)
Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G-CO 4 (page 76)
Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G-CO 5 (page 76)
Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

G-CO 6 (page 76)
Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G-CO 7 (page 76)
Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G-CO 8 (page 76)
Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Geometry

| G2 | Geometric Relationships and Properties (15 days) |

Description

This unit brings together many of the "classic" theorems of geometry. The emphasis should be on the many roles of proof (a la Devilliers) and a focus on the mathematical practice of making viable arguments and critiquing the reasoning of others. Multiple representations (coordinates, sythemic, and algebraic proofs of properties can be analyzed and compared)

Congruence   G-CO
Prove geometric theorems.
G-CO 9 (page 76)
Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

G-CO 10 (page 76)
Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

G-CO 11 (page 76)
Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Circles   G-C
G-C 3 (page 77)
Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Geometry

G3   Similarity (20 days)

Description

Similarity is defined using congruence and dilation. This geometric transformation definition of similarity and congruence is more general than comparing sides and angles.
Define a similarity transformation as the composition of a dilation followed by a congruence and prove that the meaning of similarity for triangles is the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
The connection is made between similarity and linearity (why the graph of a linear function is a straight line).

   This formalizes the Grade 8 standard 8.EE 6
        Understand the connections between proportional relationships,
        lines, and linear equations.

Dilations and Similarity: Subunit 1

Similarity, Right Triangles, and Trigonometry   G-SRT
Understand similarity in terms of similarity transformations.
G-SRT 1 (page 77)
Verify experimentally the properties of dilations given by a center and a scale factor:
a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G-SRT 2 (page 77)
Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Similarity Theorems: Subunit 2

Explore properties of similarity and prove theorems involving similarity.

G-SRT 3 (page 77)
Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

G-SRT 4 (page 77)
Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

G-SRT 5 (page 77)
Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Geometry

Description

Making connections between geometric relationships and properties and algebraic expressions is a powerful mathematical tool. This unit provides opportunities to go back and forth between the algebra and geometry. It also builds upon the previous unit making connections between similarity and linearity, for example in $f(x) = mx$ the slope m can be seen as a shape invariant of a right triangle and the x can viewed as the scale factor of the dilation centered at the origin.

Coordinates were introduced in grade 5:

Grade 5 Geometry 1: Graph points on the coordinate plane to solve real-world and mathematical problems.

Expressing Geometric Properties with Equations   G-GPE
Use coordinates to prove simple geometric theorems algebraically.
G-GPE 4 (page 78)
Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

G-GPE 5 (page 78)

Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G-GPE 6 (page 78)
Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

G-GPE 7 (page 78)
Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*

Geometry

G5   Circles and Conics (15 days)

Description

This unit includes key results about circles, treated both synthetically, with attention to constructions, and analytically.

Conic sections can arise out of mechanical constructions (e.g. loop of string around two fixed nails to produce an ellipse, paper folding to produce a parabola). Applications can be considered such as elliptic orbits, parabolic motion of projectiles.

Circle Theorems: Subunit 4.1

Circles   G-C
Understand and apply theorems about circles.
G-C 1 (page 77)
Prove that all circles are similar.

G-C 2 (page 77)
Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

G-C 5 (page 78)
Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Conic Sections: Subunit 4.2

Expressing Geometric Properties with Equations   G-GPE
Translate between the geometric description and the equation for a conic section.
G-GPE 1 (page 78)
Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G-GPE 2 (page 78)
Derive the equation of a parabola given a focus and directrix.

Geometry

G6   Trigonometric Ratios (15 days)

The class of right triangles can be classified up to similarity using "within" comparisons, i.e. the ratio of the lengths of two edges in a right triangle is an important "shape invariant". These ratios have been given names: sine = opposite/hypotenuse, cosine = adjacent/hypotenuse.

Trigonometric Ratios: Subunit 5.1

Similarity, Right Triangles, and Trigonometry   G-SRT
Define trigonometric ratios and solve problems involving right triangles.
G-SRT 6 (page 77)
Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G-SRT 7 (page 77)
Explain and use the relationship between the sine and cosine of complementary angles.

G-SRT 8 (page 77)
Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

Geometry

G7   Geometric Measurement and Dimension (15 days)

Description

This unit should treat geometric measurement more comprehensively and make connections and explain the various volume formulae with attention to the structure of the algebraic expressions. Moving between 2 and 3 dimensions such as via slicing and rotation, and establishing phenomena as fundamental contexts for quadratic relationships and volume phenomena as fundamental contexts for cubic relationships. .

This builds on the 7th grade geometry standard:
7G-G Solve real-life and mathematical problems involving angle measure, area, surface area, and volume

Geometric Measurement and Dimension   G-GMD
Explain volume formulas and use them to solve problems.
G-GMD 1 (page 78)
Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

G-GMD 3 (page 78)
Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Geometric Measurement and Dimension   G-GMD
Visualize relationships between two-dimensional and three-dimensional objects.
G-GMD 4 (page 78)
Identify the shapes of two-dimensional cross-sections of three- dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Geometry

M4   Capstone Geometric Modeling Project (10 days)

Modeling with Geometry   G-MG
Apply geometric concepts in modeling situations.
G-MG 1 (page 78)
Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

G-MG 2 (page 78)
Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*

G-MG 3 (page 78)
Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

Probability and Statistics

Description

Builds upon knowledge of probability first learned in seventh grade that includes uses of random sampling to draw inferences, compares inferences between populations, investigates chance processes, and develops, uses and evaluates probability models.

The unit introduces foundational properties and rules of probabilities, starting with events as subsets of sample space and introducing independence and conditional probability. The unit includes treatment of rules of probability to determine likelihood of compound events and introduces counting principals to calculate probability.

Content:
Understand independence and conditional probability and uses them to interpret data. Describe events as subsets of a sample space. Understand independence of events A and B as P(A and B) = P(A)*P(B), conditional probability as P(A and B)/P(B) . Construct and use two-way frequency tables. Use rules of probabilities to calculate the probability of compound events. Apply the Addition Rule and interpret the answer in terms of the model. Use permutations and combinations to compute probabilities..

Conditional Probability and the Rules of Probability   S-CP
Understand independence and conditional probability and use them to interpret data.
S-CP 1 (page 82)
Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

S-CP 2 (page 82)
Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S-CP 3 (page 82)
Understand the conditional probability of *A* given *B* as $P(A$ and $B)/P(B)$, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.

S-CP 4 (page 82)
Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

Probability and Statistics

S-CP 5 (page 82)
Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Use the rules of probability to compute probabilities of compound events in a uniform probability model.
S-CP 6 (page 82)
Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A,* and interpret the answer in terms of the model.

S-CP 7 (page 82)
Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

S1   Statistics (30 days)

Description

Builds upon statistics introduced in sixth and eighth grades that includes understanding statistical variability, interpreting and describing distributions of data and understanding the association in bivariate data.

The unit begins with interpreting, summarizing and representing single variable categorical and quantitative data sets using plots, graphs, measures of center and spread. The treatment continues with two variables data sets. The unit introduces statistical tools to investigate bivariate data that include frequency tables, scatterplots, and lines that fit the data. Students interpret, describe and summarize the data to make determinations regarding linear and non-linear associations, clustering, outliers, positive, negative and no correlations. The treatment continues by using linear, quadratic and exponential functions and correlation coefficients to make inferences and solve problems in the context of data sets. The students learn to distinguish between correlation and causation.

:Subunit 1

Interpreting Categorical and Quantitative Data   S-ID
Summarize, represent, and interpret data on a single count or measurement variable.
S-ID 1 (page 81)
Represent data with plots on the real number line (dot plots, histograms, and box plots).

S-ID 2 (page 81)
Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S-ID 3 (page 81).
Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

:Subunit 2

Interpreting Categorical and Quantitative Data   S-ID
Summarize, represent, and interpret data on two categorical and quantitative variables.
S-ID 5 (page 81)
Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

S-ID 6 (page 81)
Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
S-ID 6a (page 81)
Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
S-ID 6b (page 81)
Informally assess the fit of a function by plotting and analyzing residuals.
S-ID 6c (page 81)
Fit a linear function for a scatter plot that suggests a linear association.

Probability and Statistics

Interpret linear models.
S-ID 7 (page 81)
Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

S-ID 8 (page 81)
Compute (using technology) and interpret the correlation coefficient of a linear fit.

S-ID 9 (page 81) Distinguish between correlation and causation.

Probability and Statistics

S2　Statistics (Random processes) (26 days)

Description

This unit builds on the prior high school units of probability and statistics and involves significant use of modeling. The treatment begins with understanding the random processes that underlie statistical experiments. The unit connects processes of inference from statistics with tools and techniques from probability. This includes taking random samples to make judgments about a population and determining whether a model is consistent from the empirical data retrieved. Surveys, experiments, simulations and observational studies are use to collect and study data. Students make inferences and justify conclusions from analyzing the margin of error, examining the standard deviation of distributions, comparing difference between populations, and calculating whether a difference is significant. The unit culminates with a project in which a situation is modeled, analyzed, and a concluding report is generated.

:Subunit 1

Making Inferences and Justifying Conclusions　　S-IC
Understand and evaluate random processes underlying statistical experiments.
S-IC 1(page 81)
Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S-IC 2 (page 81)
Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

Making Inferences and Justifying Conclusions　　S-IC
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
S-IC 3 (page 81)
Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

S-IC 4 (page 82)
Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

S-IC 5 (page 82)
Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

S-IC 6 (page 82)
Evaluate reports based on data.

:Subunit 2

Interpreting Categorical and Quantitative Data　　S-ID
Summarize, represent, and interpret data on a single count or measurement variable.
S-ID 4 (page 81)
Use the mean and standard deviation of a data set to fit it to a normal distribution and to   estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.