The Common Core Learning Standards Aligned to "A Story of Units"

COMMON CORE

Overview of Pre-Kindergarten Standards as Aligned to Modules

Module 1: Analyze, Sort, Classify, and Count up to 5
Module 2: Analyze, Compare, Create, and Compose Shapes
Module 3: Count and Answer "How Many" Questions up to 10
Module 4: Describe and Compare Length, Weight, and Capacity
Module 5: Write Numerals to 5, Addition and Subtraction Stories, Count to 20

Students enter pre-kindergarten and find a well planned, sequential math program awaiting, one that is embedded within hands-on, playful, interactive, largely concrete experiences. Students are encouraged to use their math words to communicate their observations.

The first step is to analyze, sort, classify, and count up to 5 with meaning (M1). Students practice their numbers up-to-five fluency as they encounter and engage with circles, rectangles, squares, and triangles. Students practice fluency with numbers to 5 while they are learning about shapes in Module 2. With numbers to 5 understood, work begins on extending "How Many" questions up to 10 (M3). The key here is to build from 5, using their fingers to support this perspective.

- 6 is 5 and 1.
- 7 is 5 and 2.
- 8 is 5 and 3, etc.

Thus, numbers 6-10 are 5 together with numbers 1-5, making the numbers to 10 familiar and manageable. Next, students measure length, weight, and capacity developing their word bank to include the language of comparison, "small, big, short and tall (length), heavy and light (weight), empty and full (capacity) while continuing to practice fluency with numbers to 10 (M4). With numbers 1-10 still developing, counting to 20 begins while addition and subtraction are initiated within classroom stories and playful contexts (M5).

Note: In the following alignment of five modules to the pre-kindergarten standards, when a cluster is referred to without a footnote, the cluster is taught in its entirety. There are at times, too, footnotes relevant to particular standards within a cluster.

Module and Approximate Number of Instructional Days	Standards
Module 1: Analyze, Sort, Classify, and Count up to 5 (45 days)	**Count to tell the number of objects.[1]** PK.CC.3 Understand the relationship between numbers and quantities to 10; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. PK.CC.4 Count to answer "how many?" questions about as many as 10 things arranged in a line, a rectangular array, or a circle, or as many as 5 things in a scattered configuration; given a number from 1–10, count out that many objects. **Sort objects and count the number of objects in each category.[2]** PK.MD.2 Sort objects into categories; count the numbers of objects in each category.
Module 2: Analyze, Compare, Create, and Compose Shapes (15 days)	**Identify and describe shapes (squares, circles, triangles, rectangles).** PK.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as top, bottom, up, down, in front of, behind, over, under, and next to. PK.G.2 Correctly name shapes regardless of size. **Analyze, compare, and sort objects.** PK.G.3 Analyze, compare, and sort two- and three-dimensional shapes and objects, in different sizes, using informal language to describe their similarities, differences, and other attributes (e.g., color, size, and shape). PK.G.4 Create and build shapes from components (e.g., sticks and clay balls).
Module 3: Count and Answer "How Many" Questions up to 10 (50 days)	**Count to tell the number of objects.** PK.CC.3 Understand the relationship between numbers and quantities to 10; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. PK.CC.4 Count to answer "how many?" questions about as many as 10 things arranged in a line, a rectangular array, or a circle, or as many as 5 things in a scattered configuration; given a number from 1–10, count out that many objects.

[1] *Within 5.*
[2] *Within 5.*

	Compare numbers.[3] PK.CC.5 Identify whether the number of objects in one group is more, less, greater than, fewer, and/or equal to the number of objects in another group, e.g., by using matching and counting strategies. PK.CC.6 Identify "first" and "last" related to order or position. **Sort objects and count the number of objects in each category.** PK.MD.2 Sort objects into categories; count the numbers of objects in each category. (Limit category counts to be less than or equal to 10.)
Module 4: Describe and Compare Length, Weight, and Capacity (35 days)	**Compare numbers.** PK.CC.5 Identify whether the number of objects in one group is more, less, greater than, fewer, and/or equal to the number of objects in another group, e.g., by using matching and counting strategies. PK.CC.6 Identify "first" and "last" related to order or position. **Describe and compare measurable attributes.** PK.MD.1 Identify measurable attributes of objects, such as length, and weight. Describe them using correct vocabulary (e.g., small, big, short, tall, empty, full, heavy, and light).
Module 5: Write Numerals to 5, Addition and Subtraction Stories, Count to 20. (35 days)	**Know number names and the count sequence.** PK.CC.1 Count to 20. PK.CC.2 Represent a number of objects with a written numeral 0–5 (with 0 representing a count of no objects). **Understand addition as adding to, and understand subtraction as taking from.** PK.OA.1 Demonstrate an understanding of addition and subtraction by using objects, fingers, and responding to practical situations (e.g., If we have 3 apples and add two more, how many apples do we have all together?). **Understand simple patterns.** PK.OA.2 Duplicate and extend (e.g., What comes next?) simple patterns using concrete objects.

[3] *PK. CC. 5 focuses here on "more," "less" and "equal to." "Than" is excluded and introduced in the context of measurement in Module 4.*

Overview of Kindergarten Standards as Aligned to Modules

Module 1: Classify and Count Numbers to 10
Module 2: Identify and Describe Shapes
Module 3: Comparison with Length, Weight, and Numbers to 10
Module 4: Number Pairs, Addition and Subtraction of Numbers to 10
Module 5: Numbers 10-20, Counting to 100 by 1 and 10
Module 6: Analyze, Compare, Create, and Compose Shapes

The same themes as pre-kindergarten also run throughout kindergarten. It, too, starts out realistically with solidifying the meaning of numbers to 10 with a focus on graphing, relationships to 5 and growth and shrinking patterns to 10 of "1 more" and "1 less" (M1).

Next, students learn to identify and describe shapes while practicing their fluency with numbers to 10 (M2).

Two of the most crucial themes of students' math experience begin with measurement in kindergarten: unit and comparison (M3). Students use different units to measure length, weight and capacity, and explore the relationship of those units. Comparison begins with developing the meaning of the word "than": "taller *than*", "shorter *than*", "heavier *than*", "longer *than*", etc. With the word "than" concretely understood, the (at least!) 8-year curriculum teaching sequence for terms "more than" and "less than" can begin (a topic that culminates in middle school with "y is 2 less than 3 times as much as x."). The terms "more" and "less" are abstract later in kindergarten because they refer to numbers ("7 is 2 more than 5") rather than concrete measurements ("Jim is taller than John."). "1 more, 2 more, 3 more" lead into the addition fact fluencies (+1, +2, +3). Comparing numbers leads to looking at the numbers *that make up* a number ("3 is less than 7. 3 and 4 make 7. "). This, in turn, leads naturally to discussions of addition and subtraction (M4).

With numbers 1-10 on firm ground, numbers 10-20 can be parsed as "10 together with a number from 1-10." "12 is 2 more than 10" (M5). Unlike the role of 5 in numbers 6-10, which loses significance as those numbers are shown in different configurations other than "5 and a number," the number 10 is special; it *is* the anchor that will eventually become the "ten" unit in the place value system. The year rounds out by beginning explorations of concepts in area: that shapes can be composed of smaller shapes (M6).

Note: In the following alignment of six modules to the kindergarten standards, when a cluster is referred to without a footnote, the cluster is taught in its entirety.

Module and Approximate Number of Instructional Days	Standards
Module 1: Classify and Count Numbers to 10 (43 days)	**Know number names and the count sequence.**[4] K.CC.1 Count to 100 by ones and by tens. K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1). K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). **Count to tell the number of objects.**[5] K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects. **Classify objects and count the number of objects in each category.** K.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.)
Module 2: Identify and Describe Shapes (7 days)	**Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders and spheres).** K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. K.G.2 Correctly name shapes regardless of their orientations or overall size. K.G.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").
Module 3: Comparison with Length, Weight	**Compare numbers.** K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.

[4] *Within 10.*
[5] *Within 10. K.CC.4d is taught in Module 6.*

and Numbers to 10 (43 days)	K.CC.7 Compare two numbers between 1 and 10 presented as written numerals. **Describe and compare measurable attributes.** K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. K.MD.2 Directly compare two objects with a measurable attribute in common to see which object has "more of"/"less of" the attribute and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*
Module 4: Number Pairs, Addition and Subtraction of Numbers to 10 (40 days)	**Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.** K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g. 5 = 2 + 3 and 5 = 4 + 1). K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings and record the answer with a drawing or equation. K.OA.5 Fluently add and subtract within 5.
Module 5: Numbers 10-20, Counting to 100 by 1 and 10 (30 days)	**Know number names and the count sequence.** K.CC.1 Count to 100 by ones and by tens. K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1). K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). **Count to tell the number of objects.**[6] K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects. **Work with numbers 11-19 to gain foundations for place value.** K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are

[6] *K.CC.4d is taught in Module 6.*

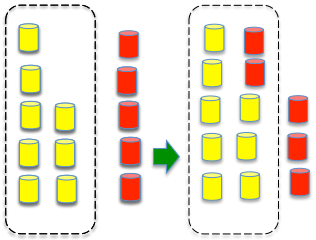
	composed of ten ones and one, two three, four, five, six, seven, eight or nine ones.
Module 6: Analyze, Compare, Create, and Compose Shapes (10 days)	**Count to tell the number of things.**[7] K.CC.4 Understand the relationship between numbers and quantities: connect counting to cardinality. d. Develop understanding of ordinal numbers (first through tenth) to describe the relative position and magnitude of whole numbers. **Analyze, compare, create and compose shapes.** K.G.4 Analyze and compare two and three dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. K.G.6 Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?*

[7] *K.CC.4d is the only standard of this cluster taught in Module 6. Ordinality is introduced in the context of constructing and manipulating shapes.*

Overview of Grade 1 Standards as Aligned to Modules

Module 1: Addition, Subtraction of Numbers to 10 and Fluency
Module 2: Place Value, Comparison, Addition and Subtraction of Numbers to 20
Module 3: Ordering and Expressing Length Measurements as Numbers
Module 4: Place Value, Comparison, Addition and Subtraction of Numbers to 40
Module 5: Identify, Compose, and Partition Shapes
Module 6: Place Value, Comparison, Addition and Subtraction of Numbers to 100

Work with "numbers to 10" continues to be a major stepping-stone in learning the place value system. Unlike pre-kindergarten and kindergarten, this year starts out with exploring addition and subtraction within 10. Fluency with addition/subtraction facts, a major gateway to later grades, also begins right away with the intention of energetically practicing the entire year (M1). The next major stepping-stone is learning to group "10 ones" as a single unit: 1 ten.



$8 + 5 = 8 + (2 + 3) = (8 + 2) + 3 = 10 + 3 = 13$

Work begins slowly by "adding and subtracting across a 10". Solutions like that shown above for 8 + 5 reinforce the need to "make 10." This strategy of the "completion of a unit" empowers students in later grades to understand the "renaming" of the addition algorithm, to add 298 and 37 (i.e., 298 + 2 + 35), and add 4 ft. 8 in. and 5 in (M2).

A module on expressing length measurement as numbers (M3) provides a few weeks in which to practice and internalize "making a 10" during daily fluency activities. Introducing measurement early also has the added bonus of opening up the variety and types of word problems that can be asked throughout the year.

The focus of adding and subtracting within 40 is on establishing "1 ten" as a new unit (M4). Before, students loosely grouped 10 objects to make 10. Now they transition to thinking of that 10 as a single unit (using 10 linker cubes stuck together, for example). Students begin to see in problems like 23+6 that they can mentally push the "2 tens" in 23 over to the side and concentrate on the familiar addition problem 3+6.

The focus of the "adding and subtracting within 100" module (M6) is different than the "within 10" and "within 40" modules. Here the new level of complexity is to also introduce the addition and subtraction algorithms using simple examples and the familiar units of 10 made out of linker cubes.

Placed in between the two heavy-duty number modules is a module on geometry (M5). The geometry module puts necessary internalization time between the "within 40" module (M4) and the "within 100" module (M6). It also gives students who may be more spatially oriented a chance to build confidence before heading back into arithmetic.

Note: In the following alignment of six modules to the first grade standards, when a cluster is referred to without a footnote, the cluster is taught in its entirety.

Module and Approximate Number of Instructional Days	Standards
Module 1: Addition, Subtraction of Numbers to 10 and Fluency (45 days)	**Represent and solve problems involving addition and subtraction.**[8] 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, e.g., by using objects, drawings and equations with a symbol for the unknown number to represent the problem. 1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **Understand and apply properties of operations and the relationship between addition and subtraction.**[9] 1.OA.3 Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)* 1.OA.4 Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8. Add and subtract within 20.* **Add and subtract within 20.**[10] 1.OA.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). 1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g. 8 + 6= 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13- 3 – 1 = 10 – 1 = 9);

[8] *Within 10.*
[9] *Within 10.*
[10] *Within 10.*

	using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4) and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 = 6 = 1 = 12 + 1 = 13.) **Work with addition and subtraction equations.**[11] 1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2. 1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ? - 3, 6 + 6 = ?.* **Measure lengths indirectly and by iterating length units.**[12] 1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*
Module 2: Place Value, Comparison, Addition and Subtraction of Numbers to 20 (35 days)	**Represent and solve problems involving addition and subtraction.** 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, e.g., by using objects, drawings and equations with a symbol for the unknown number to represent the problem. 1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **Understand and apply properties of operations and the relationship between addition and subtraction.** 1.OA.3 Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)* 1.OA.4 Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8. Add and subtract within 20.* **Add and Subtract within 20.** 1.OA.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). 1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g. 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13- 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 = 6 = 1 = 12 + 1 = 13.)

[11] *Within 10.*
[12] *1.MD.1 is taught in Module 3.*

	Work with addition and subtraction equations. 1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2. 1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ? - 3, 6 + 6 = ?.* **Understand place value.**[13] 1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones – called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight or nine ones. 1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.
Module 3: Ordering and Expressing Length Measurements as Numbers (15 days)	**Measure length indirectly and by iterating length units.** 1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object. 1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* **Represent and interpret data.** 1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
Module 4: Place Value, Comparison, Addition and Subtraction of Numbers to 40 (35 days)	**Extend the counting sequence.**[14] 1.NBT.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. **Understand place value.**[15] 1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). 1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the

[13] *Focus on numbers to 20. 1.NBT.2c is taught in Modules 4 and 6.*
[14] *Focus on numbers to 40.*
[15] *Focus on numbers to 40.*

	symbols >, =, and <. **Use place value understanding and properties of operations to add and subtract.**[16] 1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. 1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. 1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
Module 5: Identify, Compose, and Partition Shapes (15 days)	**Tell and write time and money.**[17] 1.MD.3 Tell and write time in hours and half-hours using analog and digital clocks. Recognize and identify coins, their names and their value. **Reason with shapes and their attributes.** 1.G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. 1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. 1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves, fourths*, and *quarters*, and use the phrases *half of, fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.
Module 6: Place Value, Comparison, Addition and Subtraction of Numbers to 100 (35 days)	**Extend the counting sequence.** 1.NBT.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. **Understand place value.** 1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). 1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the

[16] *Focus on numbers to 40.*
[17] *Focus on time. Coins are addressed in Module 6.*

	symbols >, =, and <. **Use place value understanding and properties of operations to add and subtract.** 1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. 1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count: explain the reasoning used. 1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **Tell and write time and money.**[18] 1.MD.3 Tell and write time in hours and half-hours using analog and digital clocks. Recognize and identify coins, their names and their value.

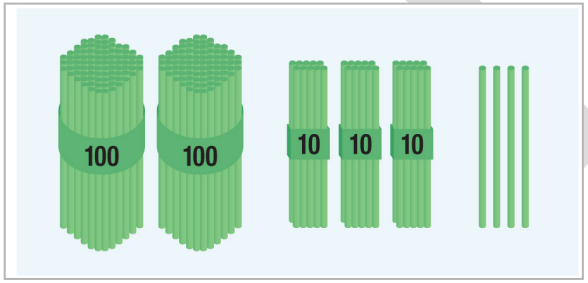
[18] *Focus on money.*

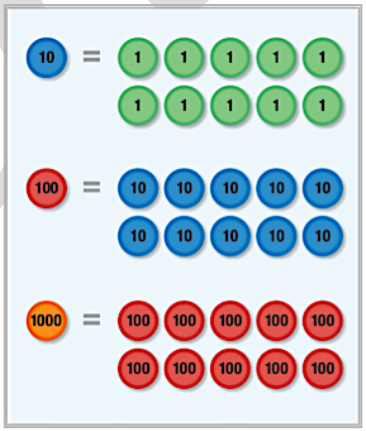
Overview of Grade 2 Standards as Aligned to Modules

Module 1: Add/Subtract Numbers to 100, Fluency with Sums and Differences to 20
Module 2: Addition and Subtraction of Length, Weight, Capacity, and Time Measurements
Module 3: Place Value, Counting and Comparison of Numbers to 1000
Module 4: Addition and Subtraction of Numbers to 1000
Module 5: Preparation for Multiplication and Division Facts
Module 6: Comparison, Addition and Subtraction with Length and Money
Module 7: Recognizing Angles, Faces, and Vertices of Shapes, Fractions of Shapes

Students arrive in grade 2 having an extensive background working with numbers to 10. This year starts with establishing a motivating, differentiated fluency program in the first few weeks that will provide the amount of practice necessary for every student to reach mastery of the addition and subtraction facts to 20 (M1). Students next learn to measure using non-standard units (while continuing to practice fluency). Like the measurement module in grade 1, this module (M2) provides the necessary background to ask varied and multifaceted measurement problems throughout the year. The major underlying goal of the measurement module, however, is for students to learn the meaning of the word "unit," essentially by employing it repeatedly in describing length units, weight units, and capacity units. The idea of a unit is the most powerful concept in PK-5 mathematics—it is the unifying theme behind all explanations in arithmetic, measurement, and geometry in elementary school.

In particular, units play a central role in the addition and subtraction algorithms of the next module (M3). All arithmetic algorithms are manipulations of *place value units*: ones, tens, hundreds, etc. In grade 2 the place value units move from a proportional model (pictured below to the left) to a non-proportional "number disk" model (pictured to the right). The place value table with number disks has the versatility to be used through grade 5 for modeling very large numbers and decimals, allowing students greater facility with and understanding of mental math and the algorithms.





The work with units continues into the next module on multiplication as well (M4). Making groups of 4 apples each establishes the unit "4 apples" (or just four) that can then be counted: 1 four, 2 fours, 3 fours, etc. Relating the new unit to the one used to create it develops the idea of multiplication: 3 groups of 4 apples equal 12 apples (or 3 fours is 12). The next module (M5) gives students another chance to practice their algorithms and problem solving skills with the most famous and most interesting units of all: dollars, dimes, and pennies.

The last module (M6) summarizes yearlong fluency work with telling time by pointing out two important relationships: an analog clock face is a "curved number line" (the precursor of a protractor) and that fractions naturally occur on a clock face (e.g. half past the hour).

Note: In the following alignment of six modules to the second grade standards, when a cluster is referred to without a footnote, the cluster is taught in its entirety.

Module and Approximate Number of Instructional Days	Standards
Module 1: Add/Subtract Numbers to 100, Fluency with Sums and Differences to 20 (10 days)	**Represent and solve problems involving addition and subtraction.**[19] 2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. **Add and subtract within 20.**[20] 2.OA.2 Fluently add and subtract within 20 using mental strategies. By end of grade 2, know from memory all sums of two one-digit numbers. **Use place value understanding and properties of operations to add and subtract.**[21] 2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

[19] *Story problems focus primarily on the positions of result and change unknown.*
[20] *From this point forward, fluency practice with addition and subtraction to 20 is part of the students' on-going experience.*
[21] *2.NBT.6, NBT.7, NBT.8 and NBT.9 are taught in Module 3.*

Module 2: Addition and Subtraction of Length, Weight, Capacity, and Time Measurements (20 days)	**Measure and estimate lengths in standard units.**[22] 2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks and measuring tapes. 2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. 2.MD.3 Estimate lengths using units of inches, feet, centimeters and meters. 2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. **Relate addition and subtraction to length.** 2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. 2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and *represent whole-number sums and differences within 100 on a number line diagram.* **Work with time and money.**[23] 2.MD.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
Module 3: Place Value, Counting and Comparison of Numbers to 1000 (25 days)	**Understand place value.** 2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens and ones; e.g., 706 equals 7 hundreds, 0 tens and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens – called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). 2.NBT.2 Count within 1000; skip-count by 5s[24], 10s, and 100s. 2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. 2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
Module 4: Addition, and	**Represent and solve problems involving addition and subtraction.**[25] 2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a

[22] *Focus on meters and centimeters in preparation for Module 3's place value.*
[23] *Focus on time. 2.MD.8 (money) is taught in Module 5.*
[24] *Use the clock to provide context for 2.MD.7.*
[25] *Story problems focus primarily on the positions of result and change unknown.*

Subtraction of Numbers to 1000: (35 days)	symbol for the unknown number to represent the problem. **Use place value understanding and properties of operations to add and subtract.** 2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. 2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations. 2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. 2.NBT.8 Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900. 2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations.
Module 5: Preparation for Multiplication and Division Facts (40 days)	**Work with equal groups of objects to gain foundations for multiplication.** 2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s: write an equation to express an even number as a sum of two equal addends. 2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. **Reason with shapes and their attributes.**[26] 2.G.2 Partition a rectangle into rows and columns of same size squares and count to find the total number of them.
Module 6: Comparison, Addition and Subtraction with Length and Money (30 days)	**Measure and estimate lengths in standard units.** 2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks and measuring tapes. 2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. 2.MD.3 Estimate lengths using units of inches, feet, centimeters and meters. 2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. **Relate addition and subtraction to length.** 2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problems. 2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

[26] *2.G.2 is taught before G.1 and G.3 because the array model is so important to the foundation for multiplication.*

	Work with time and money.[27] 2.MD.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have? **Represent and interpret data.** 2.MD.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. 2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.
Module 7: Recognizing Angles, Faces, and Vertices of Shapes, Fractions of Shapes (20 days)	**Reason with shapes and their attributes.**[28] 2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. 2.G.3 Partition circles and rectangles into two, three or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

[27] *Focus on money. Time is taught in Modules 3 and 6.*
[28] *Be sure to revisit the analog clock as part of work with 2.G.3 as an excellent application of partitioning the whole into halves, etc. and to the corresponding angle sizes.*

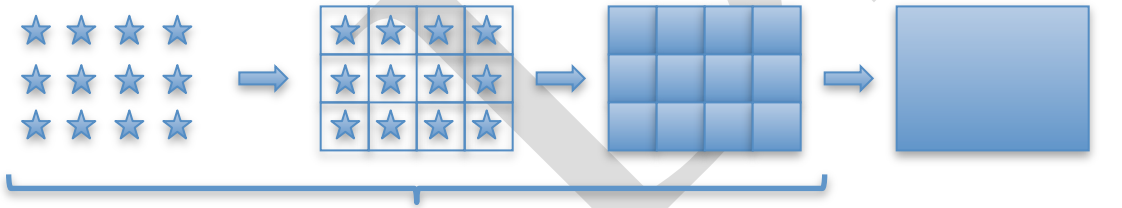
Overview of Grade 3 Standards as Aligned to Modules

Module 1: Multiplication and Division with Factors 2, 3, 4, 5 and 10
Module 2: Problem Solving with Mass, Time, and Capacity
Module 3: Multiplication and Division with Factors 6, 7, 8 and 9
Module 4: Multiplication and Area
Module 5: Fractions as Numbers on the Number Line
Module 6: Collecting and Displaying Data
Module 7: Quadrilaterals and More Challenging Word Problems

The first module (M1) builds upon multiplicative thinking with units started in grade 2. Students concentrate on the meaning of multiplication and division and begin fluency for learning products involving factors of 2, 3, 4, 5, and 10. The restricted set of facts makes learning manageable for beginners while providing enough examples to start word and measurement problems involving weight, capacity, and time in the second module (M2). The measurement module again plays the role of providing students with "internalization time" before students start into the remaining facts in the third module.

The second module also has students work with place value, comparison and rounding. The goal is to develop students' number sense well enough to build proportional bar diagrams used in solving word problems throughout third grade and beyond. "If this bar represents 62 kg, then a bar representing 35 kg needs to be slightly longer than half the 62 kg bar…"

The "2, 3, 4, 5 and 10 facts" module (M1) and the "6, 7, 8 and 9 facts" module (M3) both play key roles in preparing students to learn about area. Students often find it difficult to distinguish the different squares in a rectangular array area model, count them, and recognize that the count is related to multiplication until they have worked extensively with a Rekenrek and/or pictures of rectangular arrays involving objects only (stars, disks, etc.).



Module 1 and Module 3

Module 4: Multiplication and Area

Both modules provide important, sustained, work with both concrete and pictorial models to prepare students for area in the fourth module.

Area is the number of *area units* in a given shape (M4). When that shape is a rectangle with whole number side lengths, it is easy to partition the rectangle into squares with equal areas (like the 3rd rectangle above). The area of each square is then a fraction of the area of the rectangle, which links the fifth module to the fourth module and to the last module of grade 2 (as well as the last module in kindergarten). The goal of the fourth module, of course, is for students to transition from thinking of fractions as parts of a figure to points on a number line (M5). To make that jump students once again have to think of fractions as special types of units: Forming fractional units is exactly the same as what was done for multiplication, but the "group" is now allowed to be the amount when a whole unit is subdivided equally: "1 fourth" is the length of a segment on the number line such that the length of 4 concatenated fourth segments on the line equals 1. Once the unit "1 fourth" has been established, counting them is as easy as counting whole numbers: 1 fourth, 2 fourths, 3 fourths, 4 fourths, 5 fourths, etc.

Next, students leave the world of exact measurements behind (M6): they now are asked to estimate lengths to the nearest halves and fourths of an inch and record that information in bar graphs and line plots. This module also prepares students for the multiplicative comparison problems of grade 4 by asking students "how many more" and "how many less" questions of scaled bar graphs.

By the end of the year, students have had enough work with both linear and area measurement models to begin to study the (non) relationship between the perimeter and area of a figure, one of the concepts of the last module (M7). The year rounds out with plenty of time to solve word problems and practice fluency for concepts and skills initiated earlier in the year.

Note: In the following alignment of seven modules to the third grade standards, when a cluster is referred to without a footnote, the cluster is taught in its entirety.

Module and Approximate Number of Instructional Days	Standards
Module 1: Multiplication and Division with Factors of 2, 3, 4, 5, and 10 (25 days)	**Represent and solve problems involving multiplication and division.**[29] 3.OA.1 Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 x 7.* 3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 3.OA.4 Determine the unknown number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5= _ ÷ 3, 6 x 6=?* **Understand properties of multiplication and the relationship between multiplication and division.**[30] 3.OA.5 Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* 3.OA.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8. Multiply and divide within 100.* **Multiply and divide within 100.**[31] 3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **Solve problems involving the four operations, and identify and explain patterns in arithmetic.**[32] 3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. 3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties

[29] *Limited to factors of 2, 3, 4, 5, and 10 and the corresponding dividends.*
[30] *Limited to factors of 2, 3, 4, 5, and 10 and the corresponding dividends.*
[31] *Limited to factors of 2, 3, 4, 5, and 10 and the corresponding dividends.*
[32] *Limited to factors of 2, 3, 4, 5, and 10 and the corresponding dividends.*

	of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*
Module 2: Problem Solving with Mass, Time and Capacity (25 days)	**Use place value understanding and properties of operations to perform multi-digit arithmetic.**[33] 3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100. 3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. **Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.** 3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line. 3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.
Module 3: Multiplication and Division with Factors of 6, 7, 8, and 9. (25 days)	**Represent and solve problems involving multiplication and division.** 3.OA.1 Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 x 7.* 3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 3.OA.4 Determine the unknown number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5=____ ÷ 3, 6 x 6=?* **Understand properties of multiplication and the relationship between multiplication and division.** 3.OA.5 Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* 3.OA.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8. Multiply and divide within 100.* **Multiply and divide within 100.**[34] 3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g.,

[33] *3.NBT.3 is taught in Module 3.*
[34] *From this point forward, fluency practice with multiplication and division facts is part of the students' on-going experience.*

	knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **Solve problems involving the four operations, and identify and explain patterns in arithmetic.**[35] 3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. 3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* **Use place value understanding and properties of operations to perform multi-digit arithmetic.**[36] 3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.
Module 4: Multiplication and Area (20 days)	**Geometric measurement: understand concepts of area and relate area to multiplication and to addition.** 3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. 3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft., and improvised units). 3.MD.7 Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.
Module 5: Fractions as Numbers on the Number Line (35 days)	**Develop understanding of fractions as numbers.** 3.NF.1 Understand a fraction 1/*b* as the quantity formed by 1 part when *a* whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by a parts of size 1/*b*. 3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the

[35] *After being fully taught in Module 3, this standard (as well as OA.3) continues being practiced throughout the remainder of the school year.*
[36] *3.NBT.1 and 3.NBT.2 are taught in Module 2.*

	number line. b. Represent a fraction *a/b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line. 3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **Reason with shapes and their attributes.**[37] 3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area and describe the area of each part as ¼ of the area of the shape.*
Module 6: Collecting and Displaying Data (10 days) State Assessment	**Represent and interpret data.** 3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two- step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* 3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units--whole numbers, halves, or quarters.
Module 7: Quadrilaterals and More Challenging Word Problems[38] (40 days)	**Represent and solve problems involving multiplication and division.** 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent. **Solve problems involving the four operations, and identify and explain patterns in arithmetic.** 3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Represent and Interpret Data** 3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters.

[37] *3.G.1 is taught in Module 7.*
[38] *The seemingly eclectic set of standards in Module 7 allows for a new level of challenging word problems, as promised by the module title.*

	Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. 3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. **Reason with shapes and their attributes.** 3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Overview of Grade 4 Standards as Aligned to Modules

Module 1: Place Value, Rounding, Fluency with Addition and Subtraction Algorithms of Whole Numbers
Module 2: Unit Conversions: Addition and Subtraction of Length, Weight, and Capacity
Module 3: Multiplication and Division of Up to a 4-Digit Number by Up to a 1-Digit Number Using Place Value
Module 4: Addition and Subtraction of Angle Measurement of Planar Figures
Module 5: Order and Operations with Fractions
Module 6: Decimal Fractions
Module 7: Exploring Multiplication

In today's world, "big units" are quite common in our daily lives. For example, movies take about a gigabyte (1,000,000,000 bytes) to store on a computer while songs take about 1 megabyte (1,000,000 bytes). To understand these big numbers, the students rely upon previous mastery of rounding and the addition and subtraction algorithms. In a sense the algorithms have come full circle: In grades 2 and 3 the algorithms were the *abstract* idea students were trying to learn, but by grade 4 the algorithms have become the *concrete* knowledge students are relying upon to understand new ideas (M1). The algorithms continue to play a part in the next module (M2) on unit conversions. This module is intentionally designed to be repetitive to help students draw similarities between:

10 ones = 1 ten

100 ones = 1 hundred
100 cm = 1 m

1000 ones = 1 thousand
1000 m = 1 km
1000 g = 1 kg
1000 mL = 1 L.

Measurement problems again act as the "glue" that binds knowledge of the algorithms, mental math, place value, and real-world applications together into a coherent whole.

In the next module (M3), compound measurement units help provide the concrete foundation behind the distributive property in the multiplication algorithm: 4 × (1 m 2 cm) can be made physical using ribbon where it is easy to see the 4 copies of 1 m and the 4 copies of 2 cm. Likewise, 4 × (1 ten 2 ones) = 4 tens 8 ones.

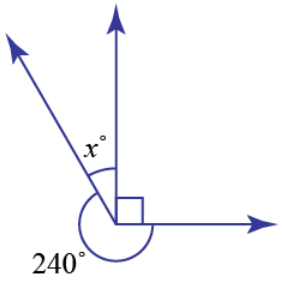
There are two pathways students travel in this curriculum to prepare for algebra in middle school. Both are extremely important. The first is solving word problems using bar diagrams (pictorial algebra). The second is solving unknown angle problems, which are

first introduced in the fourth module (M4). Students learn how to measure angles in degrees using a protractor. They also learn basic facts about angles:

1. vertical angles are equal,
2. the sum of angle measurements on a line is 180 degrees, and
3. the sum of angle measurements around a point is 360 degrees.

Armed with just these three facts (and the obvious one that angle measures of adjoining angles add), students are able to generate and solve equations that make sense:

Find the unknown angle x.



$$x + 240 + 90 = 360,$$
$$x + 330 = 360,$$
$$x = 30.$$

Geometry is the key that unlocks algebra for students *because it is visual*. The x clearly stands for a specific number: If a student wanted to, he or she could place a protractor down on that angle and measure it to find x. But doing so destroys the joy of solving the puzzle and deducing the answer for themselves.

We use fractions (M4) when there is a given unit, the *whole unit*, but we want to measure using a smaller unit, called the *fractional unit*. Students have been carefully exposed to many small units up to this point in the year:

360 degrees in 1 complete turn,
1000 g in 1 kilogram,
1000 mL in 1 liter, etc.

The beauty of fractional units is that, once defined, they behave just like whole number units:

- "3 fourths + 5 fourths = 8 fourths" like "3 apples + 5 apples = 8 apples," and
- "3 fourths × 4 = 12 fourths" like "3 apples × 4 = 12 apples."

Decimals (M5) start with the realization that decimal place value units are just special fractional units: 1 tenth = 1/10, 1 hundredth = 1/100, etc. Fluency plays an important role in both of these topics as students learn to relate 3/10 = 0.3 = 3 tenths.

The year ends with an exploratory module on multiplication (M6). Students have been practicing the algorithm for multiplying by a 1-digit number since the third module. The goal here is to structure opportunities for them to "discover" ways to multiply 2-digit × 2-digit numbers by using their tools (place value tables, area models, bar models, number disks, the distributive property and equations, etc.).

Note: In the following alignment of seven modules to the fourth grade standards, when a cluster is referred to without a footnote, the cluster is taught in its entirety.

Module and Approximate Number of Instructional Days	Standards
Module 1: Place Value, Rounding, Fluency with Addition and Subtraction Algorithms of Whole Numbers (25 days)	**Use the four operations with whole numbers to solve problems.**[39] 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Generalize place value understanding for multi-digit whole numbers.** 4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* 4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. 4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place. **Use place value understanding and properties of operations to perform multi-digit arithmetic.**[40] 4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

[39] *4.OA.1 and 4.OA.2 are taught in Module 3.*
[40] *4.NBT.5 and 4.NBT.6 are taught in Module 3.*

Module 2: Unit Conversions: Addition and Subtraction of Length, Weight, and Capacity (7 days)	**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**[41] 4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft. is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …* 4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects and money including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
Module 3: Multiplication and Division of Up to a 4-Digit Number by Up to a 1-Digit Number Using Place Value (43 days)	**Use the four operations with whole numbers to solve problems.** 4.OA.1 Interpret a multiplication equation as a comparison, e.g. interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparison as multiplication equations. 4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Gain familiarity with factors and multiplies.** 4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**[42] 4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of*

[41] *The focus of this module is on the metric system to reinforce place value, compound units, and word problems with unit conversions. Decimal and fraction word problems wait until Modules 4 and 6. 4.MD.3 is taught in Module 3.*
[42] *4.MD.1 and 4.MD.2 are taught in Modules 2 and 5.*

	a rectangular room given the area of the flooring and the length, by viewing the area formula and the length, by viewing the area formula as a multiplication equation with an unknown factor.
Module 4: Addition and Subtraction of Angle Measurements of Planar Figures (20 days)	**Geometric measurements: understand concepts of angle and measure angles.** 4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. 4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. 4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. **Draw and identify lines and angles, and classify shapes by properties of their lines and angles.** 4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. 4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. 4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
Module 5: Order and Operations with Fractions (45 days)[43]	**Extend understanding of fraction equivalence and ordering.** 4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. 4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.** 4.NF.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 +*

[43] *Tenths and hundredths are important fractions in this module, represented in decimal form in Module 6.*

State Assessment	*1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. 4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a/b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* b. Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* **Represent and interpret data.** 4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*
Module 6: Decimal Fractions (20 days)	**Understand decimal notations for fractions, and compare decimal fractions.**[44] 4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* 4.NF.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* 4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.
Module 7: Exploring Multiplication (20 days)	**Use the four operations with whole numbers to solve problems.** 4.OA.1 Interpret a multiplication equation as a comparison, e.g. Interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparison as multiplication equations. 4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including

[44] *In this module we continue to work with fractions, now including decimal form.*

	rounding. **Use place value understanding and properties of operations to perform multi-digit arithmetic.**[45] 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**[46] 4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft. is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …* 4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

[45] *In Module 7, the focus is on multiplying two two-digit numbers.*
[46] *The focus now is on customary units in word problems for application of fraction concepts.*

Overview of Grade 5 Standards as Aligned to Modules

Module 1: Whole Number and Decimal Fraction Place Value to the the One Thousandths
Module 2: Multi-Digit Whole Number and Decimal Fraction Operations
Module 3: Addition, Subtraction, Multiplication and Division of Fractions
Module 4: Extensions and Applications of Multiplication and Division of Fractions and Decimal Fractions
Module 5: Addition and Multiplication with Volume and Area
Module 6: Graph Points on the Coordinate Plane to Solve Problems

Students' experiences with the algorithms as ways to manipulate place value units in grades 2-4 really begins to pay dividends in grade 5. Whole number patterns with number disks on the place value table are easily generalized to decimal numbers (M1). As students work word problems with measurements in the metric system, where the same patterns occur, they begin to appreciate the value and the meaning of decimals. Fractions of the form 1/10, 1/100, 1/1000 also play a prominent role in the first module and are used in investigating patterns on the place value table.

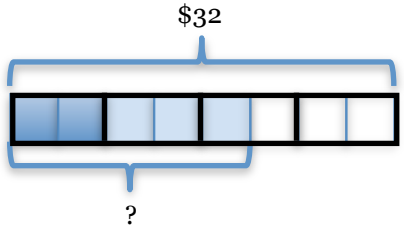
The second module (M2) starts with giving students a chance to practice and hone their skills at multiplying and dividing (decimal) numbers by 1-digit whole numbers. They are now ready to generalize the 1-digit algorithms to the multi-digit whole number versions (multi-digit *decimal* multiplication such as 4.1 × 3.4 and division such as 4.5 ÷ 1.5 are studied in Module 4). For multiplication, students must grapple with and fully understand the distributive property (one of the key reasons for *teaching* the multi-digit algorithm). While the multi-digit multiplication algorithm is a straightforward generalization of the 1-digit multiplication algorithm, the division algorithm with 2-digit divisor requires far more care to teach because students have to also learn estimation strategies, error correction strategies, and the idea of successive approximation (all of which are central concepts in math, science, and engineering).

The work with place value units in the first two modules paves the path to fraction units and arithmetic with fractions (M3), for example,

> "8 ninths ÷ 2 = 4 ninths" is just like what was learned earlier, "8 tenths ÷ 2 = 4 tenths."

A new level of complexity in the third module (M3) and fourth module (M4) is relating different fractional units to a common fractional unit: 1 third + 1 fourth = 4 twelfths + 3 twelfths = 7 twelfths. Relating different fractional units together back to the whole unit requires extensive work with area and number line models, fluency, and bar diagrams used in word problems. Bar diagrams start in this curriculum in the early grades and grow in power and usefulness as students progress through the grades. At the heart of a bar diagram is again the idea of *forming units*. In fact, forming units to solve word problems is one of the most powerful examples of the unit theme and is particularly helpful for understanding fraction arithmetic, as in the following example:

Jill had \$32. She gave $\frac{1}{4}$ of her money to charity and $\frac{3}{8}$ of her money to her brother. How much did she give altogether?



Solution with units:

8 units = \$32
1 unit = \$4
5 units = \$20.

with arithmetic:

$\frac{1}{4}+\frac{3}{8}=\frac{2}{8}+\frac{3}{8}=\frac{5}{8}$

$\frac{5}{8}\times 32 = 20.$

Jill gave \$20 altogether.

Near the end of Module 4 students have the insight needed with fractions and whole numbers operations to begin to explore multdigit decimal multiplication and division. In multiplying 2.1 × 3.8, for example, students have multiple strategies they can rely on to locate the decimal point in the final answer, including:

- unit awareness: 2.1 × 3.8 = 21 tenths × 38 tenths = 798 hundredths,
- estimation (through rounding): 2.1 × 3.8 ≈ 2 × 4 = 8, so 2.1 × 3.8 = 7.98,
- fraction multiplication: 21/10 × 38/10 = (21 × 38)/(10 × 10).

Similar strategies enrich students understanding of division and help them to see multidigit decimal division as "whole number division in a different unit." For example, we divide to find, "How many groups of 3 apples are there in 45 apples?" and write 45 apples ÷ 3 apples = 15. Similarly, 4.5 ÷ 0.3 can be written as "45 tenths ÷ 3 tenths" with the same answer: there are 15 groups of 0.3 in 4.5. The same idea was used to introduce fraction division earlier in the module, thus gluing division with whole numbers, fractions, and decimals together through an understanding of units.

The fraction module prepares students through the daily use of area models for an in depth discussion of area in the next module (M5). But the module on area and volume also reinforces the work done in the fraction module: questions can now be asked about how the area changes when a rectangle is scaled by a whole or fractional scale factor. Measuring volume once again highlights the unit theme as a unit cube is chosen to represent a volume unit and used to measure the volume of simple shapes made out of rectangular prisms.

Scaling is returned to in the last module on coordinate plane (M6). Ever since the growth and shrinking patterns were first introduced in kindergarten, students have been using bar graphs to display data and patterns. All that work with bar graphs over the years has set the stage for line plots, which is both the natural extension of bar graphs and the precursor to linear functions. It is in

this final module of K-5 that a simple line plot of a straight line is presented on a coordinate plane and students are asked about the scaling relationship between the increase in the units of the vertical axis for 1 unit of increase in the horizontal axis—the first hint of slope and the beginning of the "Story of Ratios" in middle school.

Note: In the following alignment of six modules to the fifth grade standards, when a cluster is referred to without a footnote, the cluster is taught in its entirety.

Module and Approximate Number of Instructional Days	Standards
Module 1: Whole Number and Decimal Fraction Place Value to the the One Thousandths (20 days)	**Understand the place value system.** 5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. 5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.[47] 5.NBT.3 Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347 . 392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. 5.NBT.4 Use place value understanding to round decimals to any place. **Convert like measurement units within a given measurement system.**[48] 5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.
Module 2: Multi-Digit Whole Number and Decimal Fraction Operations (35 days)	**Write and interpret numerical expressions.** 5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. 5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.* **Understand the place value system.**[49] 5.NBT.1 Recognize that in a multi-digit number, a digit in the ones place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. 5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

[47] *The focus in this module is on patterns when multiplying or dividing by 10, not powers of 10.*
[48] *The focus of this module is on the metric system to reinforce place value and its compound units.*
[49] *These standards, taught in Module 1 in the context of place value, are now taught in the context of the operations with whole numbers.*

	Perform operations with multi-digit whole numbers and with decimals to hundredths.[50] 5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm. 5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
Module 3: Addition, Subtraction, Multiplication and Division of Fractions (30 days)	**Write and interpret numerical expressions.**[51] 5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. 5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.* **Use equivalent fractions as a strategy to add and subtract fractions.**[52] 5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* 5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.* **Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**[53] 5.NF.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

[50] *Focus on decimal multiplication of a single-digit, whole number factor times a multi digit number with up to 2 decimal places (e.g. 3 x64.98). Restrict decimal division to a single digit whole number divisor with a multi digit dividend with up to 2 decimal places. (e.g. 64.98 ÷ 3). The balance of the standard is taught in Module 4.*
[51] *These skills are applied to fractions in this module.*
[52] *Examples in this module also include tenths and hundredths in fraction and decimal form.*
[53] *The balance of this cluster is taught in Module 4.*

	Convert like measurement units within a given measurement system.[54] 5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. **Represent and interpret data.** 5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
Module 4: Extensions and Applications of Multiplication and Division of Fractions and Decimal Fractions (25 days)	**Perform operations with multi-digit whole numbers and with decimals to hundredths.**[55] 5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**[56] 5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show* $(2/3) \times 4 = 8/3$, *and create a story context for this equation. Do the same with* $(2/3) \times (4/5) = 8/15$. *(In general,* $(a/b) \times (c/d) = ac/bd$.*)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. 5.NF.5 Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. 5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. 5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story*

[54] *The focus of 5.MD.1 in this module is on the customary system of units as a means of introducing fractions (e.g. 1 inch is 1/12 foot, 1 foot is 1/3 yard, etc).*
[55] *Teach problems such as 2.7 x 2.1 and 4.5 ÷ 1.5. See "Progressions" pgs. 17 – 18 http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_nbt_2011_04_073.pdf*
[56] *The focus of 5.NF.4 in this module is only on part a. 5.NF.4b is taught in Module 5. Include problems involving decimal fractions throughout the cluster.*

	context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3. b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*
Module 5: Addition and Multiplication with Volume and Area (25 days)	**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**[57] 5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. **Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.** 5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units. 5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. 5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. **Classify two-dimensional figures into categories based on their properties.** 5.G.3 Understand that attributes belonging to a category of two- dimensional figures also belong to all subcategories of that category.

[57] *5.NF.4a is taught in Module 4. In this module 5.NF.4b is applied to multiplying to find volume and area. 5.NF.4b certainly includes decimal fraction side lengths of sides of a rectangle (in both fraction and decimal form).*

State Assessment	For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. 5.G.4 Classify two-dimensional figures in a hierarchy based on properties.
Module 6: Graph Points on the Coordinate Plane to Solve Problems (40 days)	**Write and interpret numerical expressions.** 5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.* **Analyze patterns and relationships.** 5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are* ntwice *the corresponding terms in the other sequence. Explain informally why this is so.* **Graph points on the coordinate plane to solve real-world and mathematical problems.** 5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). 5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.